Joint Entrance Exam | Mains - 2019

9th January 2019 | 9.30 AM – 12.30 PM

PHYSICS, CHEMISTRY & MATHEMATICS

***Important Instructions*:**

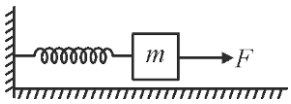
1. Immediately fill in the particulars on this page of the Test Booklet with only Black Ball Point Pen provided in the examination hall.
2. The Answer Sheet is kept inside this Test Booklet. When you are directed to open the Test Booklet, take out the Answer Sheet and fill in the particulars carefully.
3. The test is of **3 hours** duration.
4. The Test Booklet consists of **90** questions. The maximum marks are **360**.
5. There are three parts in the question paper A, B, C consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each question is allotted **4 (four)** marks for correct response.
6. Candidate will be awarded marks as stated above in instruction No. 5 for correct response of each question. $\frac{1}{4}$ (one-fourth) marks of the total marks allotted to the questions (i.e. 1 mark) will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
7. There is only one correct response for each question. Filling up more than one response in any question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 6 above.
8. For writing particulars/marking responses on *Side-1* and *Side-2* of the Answer Sheet use ***only Black Ball Point Pen*** provided in the examination hall.
9. No candidate is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, any electronic device, etc. except the Admit Card inside the examination room/hall.
10. Rough work is to be done on the space provided for this purpose in the Test Booklet only. This space is given at the bottom of each page and in **four** pages (Page **14-18**) at the end of the booklet.
11. On completion of the test, the candidate must hand over the Answer Sheet to the **Invigilator** on duty in the Room/Hall. ***However, the candidates are allowed to take away this Test Booklet with them****.*
12. The CODE for this Booklet is **B.** Make sure that the CODE printed on Side-2 of the Answer Sheet is same as that on this Booklet. Also tally the serial number of the Test Booklet and Answer Sheet are the same as that on this booklet. In case of discrepancy, the candidate should immediately report the matter to the Invigilator for replacement of both the Test Booklet and the Answer Sheet.
13. **Do not fold or make any stray mark on the Answer Sheet.**

PART-A	PHYSICS

1. There charge $+Q, q, +Q$ are placed respectively, at distance, $0, d/2$ and d from the origin, on the x-axis. If the net force experienced by $+Q$, placed at $x = 0,$ is zero, then value of q is:

(1) $-Q/2$ (B) $-Q/4$ **(3)** $+Q/2$ **(4)** $+Q/4$

2. A block of mass m, lying on a smooth horizontal surface, is attached to a spring (of negligible mass) of spring constant k. The other end of the spring is fixed, as shown in the figure. The block is initially at rest in its equilibrium position. If now the block is pulled with a constant force F, the maximum speed of the block is:



(1) $\frac{F}{\sqrt{mk}}$ **(2)** $\frac{\pi F}{\sqrt{mk}}$ **(3)** $\frac{F}{\pi\sqrt{mk}}$ **(4)** $\frac{2F}{\sqrt{mk}}$

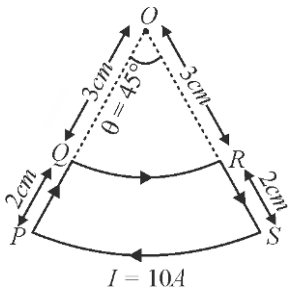
3. Drift speed of electrons, when 1.5 A of current flows in a copper wire of cross section $5\ mm^2$, is v. If the electron density in copper is $9\times10^{28}/m^3$ the value of v in mm/s is close to (Take charge of electron to be $=1.6\times10^{-19}C$)

(1) 2 **(2)** 0.02 **(3)** 3 **(4)** 0.2

4. If the angular momentum of a planet of mass m, moving around the Sun in a circular orbit is L, about the centre of the Sun, its areal velocity is:

(1) $\frac{L}{2m}$ **(2)** $\frac{4L}{m}$ **(3)** $\frac{L}{m}$ **(4)** $\frac{2L}{m}$

5. A current loop, having two circular arcs joined by two radial lines is shown in the figure. It carries a current of 10A. The magnetic field at point O will be closed to:



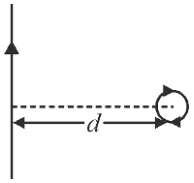
(1) $1.0\times10^{-7}T$

(2) $1.0\times10^{-5}T$

(3) $1.5\times10^{-7}T$

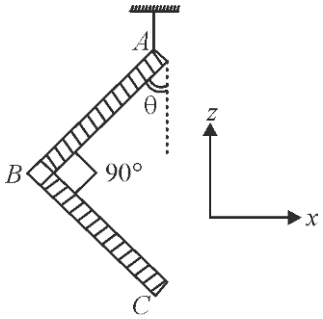
(4) $1.5\times10^{-5}T$

6. An infinitely long current carrying wire and a small current carrying loop are in the plane of the paper as shown. The radius of the loop is a and distance of its centre from the wire is $d(d >> a)$. If the loop applies a force F on the wire than:



(1) $F \propto \left(\frac{a}{d}\right)$ **(2)** $F \propto \left(\frac{a}{d}\right)^2$ **(3)** $F = 0$ **(4)** $F \propto \left(\frac{a^2}{d^3}\right)$

7. An L-shaped object, made of thin rods of uniform mass density, is suspended with a string as shown in figure. If $AB = BC$, and the angle made by AB with downward vertical is θ, then:



(1) $\tan\theta = \frac{2}{\sqrt{3}}$ **(2)** $\tan\theta = \frac{2}{2\sqrt{3}}$

(3) $\tan\theta = \frac{1}{3}$ **(4)** $\tan\theta = \frac{1}{2}$

8. A mixture of 2 moles of helium gas (atomic mass = 4u), and 1 mole of argon gas (atomic mass = 40 u) is kept at 300 K in a container. The ratio of their rms speeds $\left[\frac{V_{rms}(\text{helium})}{V_{rms}(\text{argon})}\right]$, is close to:

(1) 3.16 **(2)** 0.45 **(3)** 2.24 **(4)** 0.32

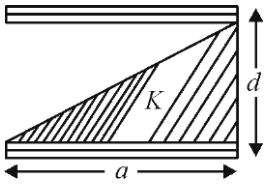
9. A plane electromagnetic wave of frequency 50 MHz travels in free space along the positive x-direction. At a particular point in space and time, $\vec{E} = 6.3\hat{j}\ V/m$. The corresponding magnetic field $\vec{B}$, at that point will be:

(1) $6.3\times10^{-8}\,\hat{k}T$ **(2)** $18.9\times10^{-8}\,\hat{k}T$ **(3)** $2.1\times10^{-8}\,\hat{k}T$ **(4)** $18.9\times10^{8}\,\hat{k}T$

10. A heavy ball is mass M is suspended from the ceiling of a car by a light string of mass $m(m << M)$. When the car is at rest, the speed of transverse waves in the string is 60ms^{-1}. When the car has acceleration a, the wave-speed increases to 60.5 ms^{-1}. The value of a, in terms of gravitational acceleration g, is closest to:

(1) $\frac{g}{30}$ **(2)** $\frac{g}{20}$ **(3)** $\frac{g}{5}$ **(4)** $\frac{g}{10}$

11. A parallel plate capacitor is made of two square plates of side 'a', separated by a distance $d(d << a)$. The lower triangular portion is filled with a dielectric of dielectric constant K, as shown in the figure. Capacitance of this capacitor is:



(1) $\frac{1}{2}\frac{K\in_0 a^2}{d}$ **(2)** $\frac{K\in_0 a^2}{d}\ln K$ **(3)** $\frac{K\in_0 a^2}{d(K-1)}\ln K$ **(4)** $\frac{K\in_0 a^2}{2d(K+1)}$

12. A rod, of length L at room temperature and uniform area of cross section A, is made of a metal having coefficient of linear coefficient of linear expansion $\alpha/°C$. It is observed that an external compressive force F, is applied on each of its ends, prevents any change in the length of the rod, when its temperature rises by ΔTK. Young's modulus, Y, for this metal is :

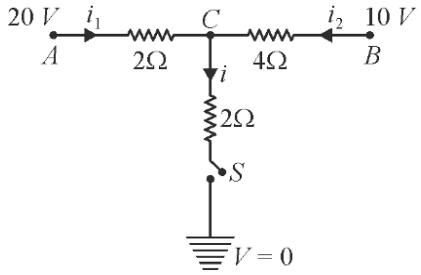
(1) $\frac{F}{2A\alpha\Delta T}$ **(2)** $\frac{F}{A\alpha(\Delta T-273)}$ **(3)** $\frac{F}{A\alpha\Delta T}$ **(4)** $\frac{2F}{A\alpha\Delta T}$

13. Surface of certain metal is first illuminated with light of wavelength $\lambda_1 = 350$ nm and then, by light of wavelength $\lambda_2 = 540$ nm. It is found that the maximum speed of the photo electrons in the two cases differ by a factor of 2. The work function of the metal (in eV) is close to:

(Energy of photon $= \frac{1240}{\lambda(\text{in nm})}eV$)

(1) 2.5 **(2)** 1.8 **(3)** 5.6 **(4)** 1.4

14. When the switch *S*, in the circuit shown, is closed, then the value of current *i* will be:



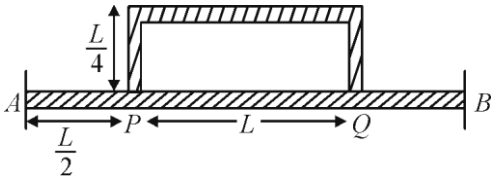
(1) 3 *A* **(2)** 4 *A* **(3)** 2 *A* **(4)** 5 *A*

15. A particle is moving with a velocity $\vec{u} = K(y\hat{i} + x\hat{j})$, where *K* is a constant. The general equation for its path is:

(1) *xy* = constant **(2)** $y^2 = x^2 + \text{constant}$

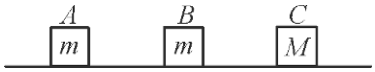
(3) $y^2 = x + \text{ constant}$ **(4)** $y = x^2 + \text{constant}$

16. Temperature difference of 120°*C* is maintained between two ends of a uniform rod *AB* of length 2*L*. Another bent rod *PQ*, of same cross-section as *AB* and length $\frac{3L}{2}$, is connected across *AB* (See figure). In steady state, temperature difference between *P* and *Q* will be close to:



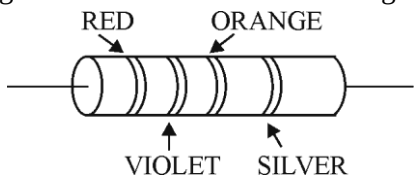
(1) 45°C **(2)** 60°C **(3)** 35°C **(4)** 75°C

17. Three blocks *A*, *B* and *C* are lying on a smooth horizontal surface, as shown in the figure. *A* and *B* have equal masses, *m* while *C* has mass *M*. Block *A* is given an initial speed v towards *B* due to which it collides with B perfectly inelastically. The combined mass collides with *C*, also perfectly inelastically $\frac{5}{6}$th of the initial kinetic energy is lost in whole process. What is value of *M*/*m*?



(1) 2 **(2)** 5 **(3)** 3 **(4)** 4

18. A resistance is shown in the figure. Its value and tolerance are given respectively by:



(1) 270Ω, 5% **(2)** 27 *k*Ω, 10% **(3)** 27 *k*Ω, 20% **(4)** 270Ω, 10%

19. Mobility of electrons in a semiconductor is defined as the ratio of their drift velocity to the applied electric field. If, for an n-type semiconductor, the density of electrons is 10^{19} m^{-3} and their mobility is 1.6 $m^2/(V.s)$ then the resistivity of the semiconductor (since it is an n-type semiconductor contribution of holes is ignored) is close to:

(1) 2Ωm **(2)** 4Ωm **(3)** 0.2Ωm **(4)** 0.4Ωm

20. Two coherent sources produce waves of different intensities which interfere. After interference, the ratio of the maximum intensity to the minimum intensity is 16. The intensity of the waves are in the ratio:

(1) 25 : 9 **(2)** 16 : 9 **(3)** 5 : 3 **(4)** 4 : 1

21. A conducting circular loop made of a thin wire, has area $3.5\times10^{-3}m^2$ and resistance 10Ω. It is placed perpendicular to a time dependent magnetic field $B(t)=(0.4T)\sin(50\pi t)$. The field is uniform in space. Then the net charge flowing through the loop during t = 0 s and t = 10 ms is close to:

(1) 7 mC **(2)** 21 mC **(3)** 14 mC **(4)** 6 mC

22. A copper wire is stretched to make it 0.5 % longer. The percentage change in its electrical resistance if its volume remains unchanged is:

(1) 2.5 % **(2)** 0.5 % **(3)** 1.0 % **(4)** 2.0 %

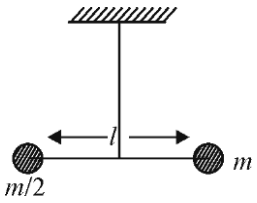
23. For a uniformly charged ring of radius *R*, the electric field on its axis has the largest magnitude at a distance h from its centre. Then value of *h* is:

(1) R **(2)** $\frac{R}{\sqrt{2}}$ **(3)** $\frac{R}{\sqrt{5}}$ **(4)** $\frac{R}{\sqrt{5}}$

24. A sample of radioactive material *A*, that has an activity of 10 mCi $(1\,Ci=3.7\times10^{10}$ decays/s), has twice the number of nuclei as another sample of a different radioactive material *B* which has an activity of 20 mCi. The correct choices for half-lives of *A* and *B* would then be respectively:

(1) 5 days and 10 days **(2)** 20 days and 10 days
(3) 10 days and 40 days **(4)** 20 days and 5 days

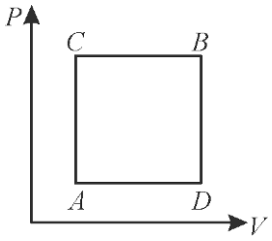
25. Two masses m and $\frac{m}{2}$ are connected at the two ends of a massless rigid rod of length *l*. The rod is suspended by a thin wire of torsional constant *k* at the centre of mass of the rod-mass system (see figure). Because of torsional constant *k*, the restoring torque is $\tau=k\theta$ for angular displacement θ. If the rod is rotated by θ_0 and released, the tension in it when it passes through its mean position will be:



(1) $\frac{3k\theta_0^2}{l}$ **(2)** $\frac{k\theta_0^2}{2l}$ **(3)** $\frac{2k\theta_0^2}{l}$ **(4)** $\frac{k\theta_0^2}{l}$

26. A gas can be taken from *A* to *B* via two different process *ACB* and *ADB*.

When path *ACB* is used 60 J of heat flows into the system and 30 J of work is done by the system. If path *ABD* is used work done by the system is 10 J. the heat flow into the system in path *ABD* is:



(1) 100 J **(2)** 40 J **(3)** 80 J **(4)** 20 J

27. A bar magnet is demagnetized by inserting it inside a solenoid of length 0.2 m, 100 turns, and carrying a current of 5.2 A. The coercivity of the bar magnet is:

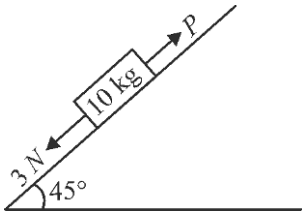
(1) 520 A/m **(2)** 285 A/m **(3)** 2600 A/m **(4)** 1200 A/m

28. A convex lens is put 10 cm from a light source and it makes a sharp image on a screen, kept 10 cm from the lens. Now a glass block (refractive index 1.5) of 1.5 cm thickness is placed in contact with the light source. To get the sharp image again, the screen is shifted by a distance *d*. Then *d* is:

(1) 1.1 cm away from the lens **(2)** 0.55 cm away from the lens

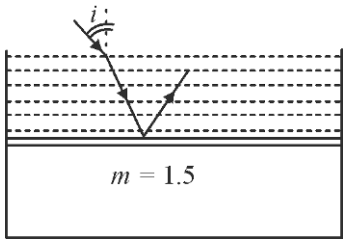
(3) 0.55 cm towards the lens **(4)** 0

29. A block of mass 10 kg is kept on a rough inclined plane as shown in the figure. A force of 3 N is applied on the block. The coefficient of static friction between the plane and the block is 0.6. What should be the minimum value of force *P*, such that the block does not move downward? (take $g = 10\ \text{ms}^{-2}$)



(1) 23 N **(2)** 32 N **(3)** 25 N **(4)** 18 N

30. Consider a tank made of glass (refractive index 1.5) with a thick bottom. It is filled with a liquid of refractive index μ. A student finds that, irrespective of what the incident angle i (see figure) is for a beam of light entering the liquid, the light reflected from the liquid glass interface is never completely polarized. For this to happen, the minimum value of μ is:



(1) $\frac{5}{\sqrt{3}}$ **(2)** $\sqrt{\frac{5}{3}}$ **(3)** $\frac{4}{3}$ **(4)** $\frac{3}{\sqrt{5}}$

PART-B	CHEMISTRY

1. Which amongst the following is the strongest acid ?

(1) CHI_3 **(2)** $CHBr_3$ **(3)** $CHCl_3$ **(4)** $CH(CN)_3$

2. Which one of the following statements regarding Henry's law is not correct ?

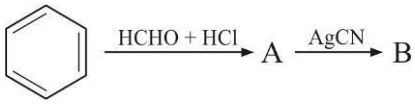
(1) Higher the value of K_H at a given pressure, higher is the solubility of the gas in the liquids

(2) The value of K_H increases with increase of temperature and K_H is function of the nature of the gas

(3) Different gases have different K_H (Henry's law constant) values at the same temperature

(4) The partial pressure of the gas in vapour phase is proportional to the mole fraction of the gas in the solution

3. The compounds A and B in the following reaction are, respectively :



(1) A = Benzyl chloride, B = Benzyl isocyanide

(2) A = Benzyl chloride, B = Benzyl cyanide

(3) A = Benzyl alcohol, B = Benzyl isocyanide

(4) A = Benzyl alcohol, B = Benzyl cyanide

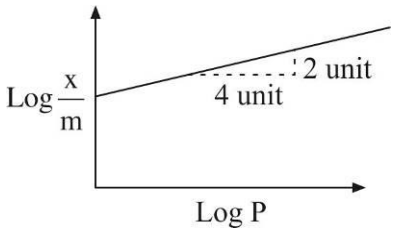
4. The anodic half-cell of lead-acid battery is recharged using electricity of 0.05 Faraday. The amount of $PbSO_4$ electrolyzed in g during the process is: (Molar mass of $PbSO_4 = 303\,g\,mol^{-1}$)

(1) 11.4 **(2)** 15.2 **(3)** 22.8 **(4)** 7.6

5. 0.5 moles of gas A and x moles of gas B exert a pressure of 200 Pa in a container of volume $10\,m^3$ at 1000K. Given R is the gas constant in $JK^{-1}mol^{-1}$, x is:

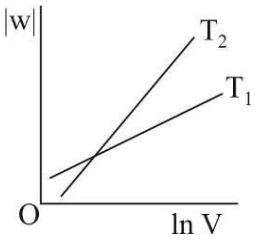
(1) $\frac{4+R}{2R}$ **(2)** $\frac{2R}{4-R}$ **(3)** $\frac{2R}{4+R}$ **(4)** $\frac{4-R}{2R}$

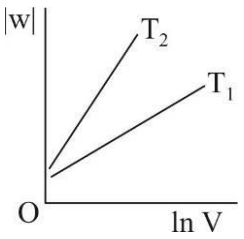
6. Adsorption of a gas follows Freundlich adsorption isotherm. In the given plot, x is the mass of the gas adsorbed on mass m of the adsorbent at pressure p. $\frac{x}{m}$ is proportional to :

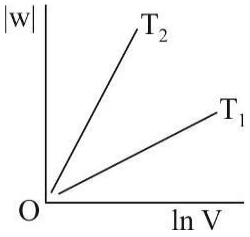


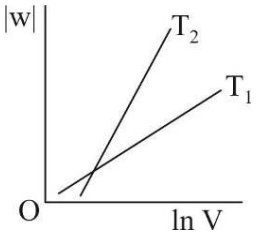
(1) $P^{1/2}$ **(2)** P **(3)** $P^{1/4}$ **(4)** P^2

7. Consider the reversible isothermal expansion of an ideal gas in a closed system at two different temperatures T_1 and T_2 $(T_1 < T_2)$. The correct graphical depiction of the dependence of work done (w) on the final volume (V) is:

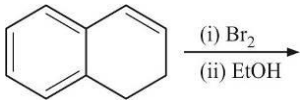
(1) 

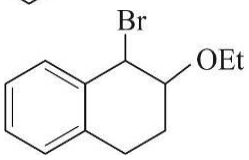
(2) 

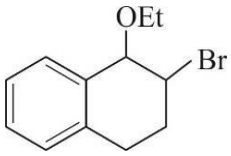
(3) 

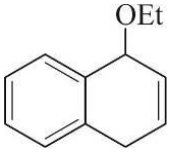
(4) 

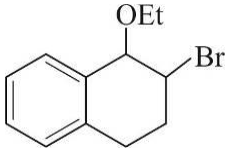
8. The major product of the following reaction is:



(1) 

(2) 

(3) 

(4) 

9. The correct decreasing order for acid strength is:

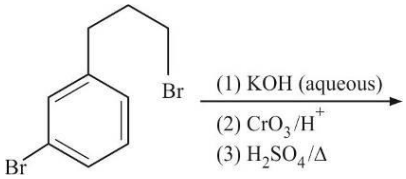
(1) $NO_2CH_2COOH > NCCH_2COOH > FCH_2COOH > ClCH_2COOH$

(2) $NO_2CH_2COOH > FCH_2COOH > CNCH_2COOH > ClCH_2COOH$

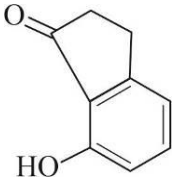
(3) $CNCH_2COOH > O_2NCH_2COOH > FCH_2COOH > ClCH_2COOH$

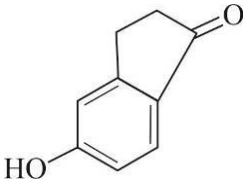
(4) $FCH_2COOH > NCCH_2COOH > NO_2CH_2COOH > ClCH_2COOH$

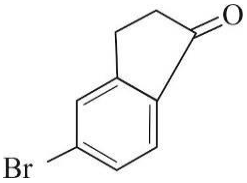
10. The major product of the following reaction is:



(1) 

(2) 

(3) 

(4) 

11. The highest value of the calculated spin-only magnetic moment (in BM) among all the transition metal complexes is:

(1) 3.87 **(2)** 4.90 **(3)** 6.93 **(4)** 5.92

12. The isotopes of hydrogen are:

(1) Protium, deuterium and tritium **(2)** Protium and deuterium only

(3) Tritium and protium only **(4)** Deuterium and tritium only

13. Aluminium is usually found in +3 oxidation state. In contrast, thallium exists in +1 and +3 oxidation states. This is due to:

(1) diagonal relationship **(2)** lanthanoid contraction

(3) inert pair effect **(4)** lattice effect

14. In general, the properties that decrease and increase down a group in the periodic table, respectively, are:

(1) electron gain enthalpy and electronegativity

(2) electronegativity and atomic radius

(3) electronegativity and electron gain enthalpy

(4) atomic radius and electronegativity

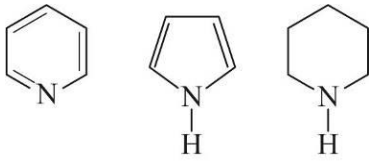
15. The following results were obtained during kinetic studies of the reaction ; $2A + B \rightarrow$ Products

Experiment	[A] (in mol L^{-1})	[B] (in mol L^{-1})	Initial Rate of reaction (in mol L^{-1} mol^{-1})
I	0.10	0.20	6.93×10^{-3}
II	0.10	0.25	6.93×10^{-3}
III	0.20	0.30	1.386×10^{-2}

The time (in minutes) required to consume half of A is:

(1) 1 **(2)** 10 **(3)** 100 **(4)** 5

16. Arrange the following amines in the decreasing order of basicity :



I II III

(1) I > II > III **(2)** III > II > I **(3)** I > III > II **(4)** III > I > II

17. The increasing order of pKa of the following amino acids in aqueous solution is:

Gly Asp Lys Arg

(1) Gly < Asp < Arg < Lys **(2)** Asp < Gly < Lys < Arg

(3) Asp < Gly < Arg < Lys **(4)** Arg < Lys < Gly < Asp

18. Correct statements among a to d regarding silicones are:

(a) They are polymers with hydrophobic character

(b) They are biocompatible

(c) In general, they have high thermal stability and low dielectric strength

(d) Usually, they are resistant to oxidation and used as greases

(1) (a), (b), (c) and (d)

(2) (a), (b) and (d) only

(3) (a) and (b) only

(4) (a), (b) and (c) only

19. The correct match between Item-I and Item-II is :

Item-I (drug)		Item-II (test)	
(A)	Chloroxylenol	**(P)**	Carbylamine test
(B)	Norethindrone	**(Q)**	Sodium hydrogen carbonate test
(C)	Sulphapyridine	**(R)**	Ferric chloride test
(D)	Penicillin	**(S)**	Bayer's test

(1) A-R; B-P; C-S; D-Q **(2)** A-Q; B-S; C-P; D-R
(3) A-Q; B-P; C-S; D-R **(4)** A-R; B-S; C-P; D-Q

20. Two complexes $[Cr(H_2O)_6]Cl_3$ (A) and $[Cr(NH_3)_6]Cl_3$ (B) are violet and yellow coloured, respectively. The incorrect statement regarding them is:

(1) both are paragenetic with three unpaired electrons
(2) Δ_0 value for (A) is less than that of (B)
(3) Δ_0 values of (A) and (B) are calculated from the energies of violet and yellow light, respectively
(4) both absorb energies corresponding to their complementary colors

21. The one that is extensively used as a piezoelectric material is:
(1) tridymite **(2)** amorphous silica **(3)** quartz **(4)** mica

22. A solution of sodium sulfate contains 92 g of Na^+ ions per kilogram of water. The molality of Na^+ ions in that solution in mol kg^{-1} is:
(1) 4 **(2)** 12 **(3)** 8 **(4)** 16

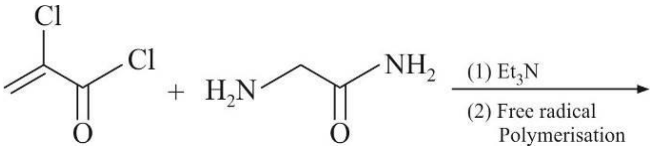
23. The alkaline earth metal nitrate that does not crystallise with water molecules, is:
(1) $Mg(NO_3)_2$ **(2)** $Sr(NO_3)_2$ **(3)** $Ba(NO_3)_2$ **(4)** $Ca(NO_3)_2$

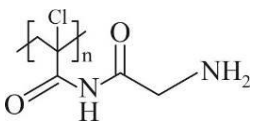
24. For emission line of atomic hydrogen form $n_i = 8$ to $n_f = n$, the plot of wave number $(\bar{\nu})$ against $\left(\frac{1}{n^2}\right)$ will be: (the Rydberg constant, R_H is in wave number unit)

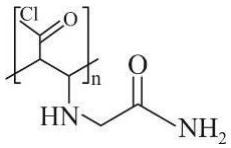
(1) Linear with slope $-R_H$ **(2)** Non linear
(3) Linear with slope R_H **(4)** Linear with intercept $-R_H$

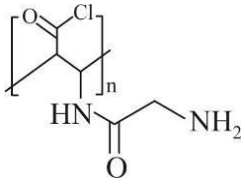
25. 20 mL of 0.1 M H_2SO_4 solution is added to 30 mL of 0.2 M NH_4OH solution. The pH of the resultant mixture is : [pk_b of $NH_4OH = 4.7$].
(1) 9.4 **(2)** 9.0 **(3)** 5.0 **(4)** 5.2

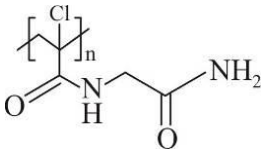
26. Major product of the following reaction is :



(1)


(2)


(3)


(4)


27. A water sample has ppm level concentration of the following metals :Fe = 0.2 ; Mn = 5.0 ; Cu = 3.0 ' Zn = 5.0. The metal that makes the water sample unsuitable for drinking is :

(1) Mn **(2)** Zn **(3)** Cu **(4)** Fe

28. According to molecular orbital theory, which of the following is true with respect to Li_2^+ and Li_2^- ?

(1) Both are stable **(2)** Li_2^+ is stable and Li_2^- is unstable

(3) Li_2^+ is unstable and Li_2^- is stable **(4)** Both are unstable

29. The major product of following reaction is :

$$R-C\equiv N \xrightarrow[(2)\ H_2O]{(1)\ AlH(i-Bu)_2} ?$$

(1) RCOOH **(2)** $RCONH_2$ **(3)** RCHO **(4)** RCH_2NH_2

30. The ore that contains both iron and copper is :

(1) malachite **(2)** azurite **(3)** copper pyrites **(4)** dolomite

PART-C	MATHEMATICS

1. If $y = y(x)$ is the solution of the differential equation, $x\frac{dy}{dx} + 2y = x^2$ satisfying $y(1) = 1$, then $y\left(\frac{1}{2}\right)$ is equal to:

(1) $\frac{7}{64}$ **(2)** $\frac{13}{16}$ **(3)** $\frac{1}{4}$ **(4)** $\frac{49}{16}$

2. If a, b and c be three distinct real numbers in G.P. and $a + b + c = xb$, then x cannot be:

(1) 2 **(2)** 4 **(3)** –3 **(4)** –2

3. If θ denotes the acute angle between the curves, $y - 10 - x^2$ and $y = 2 + x^2$ at a point of their intersection, then $|\tan\theta|$ is equal to:

(1) $\frac{8}{17}$ **(2)** $\frac{4}{9}$ **(3)** $\frac{8}{15}$ **(4)** $\frac{7}{17}$

4. Let α and β be two roots of the equation $x^2 + 2x + 2 = 0$, then $\alpha^{15} + \beta^{15}$ is equal to:

(1) –256 **(2)** –512 **(3)** 256 **(4)** 512

5. Three circles of radii $a, b, c (a < b < c)$ touch each other externally. If they have x-axis as a common tangent, then:

(1) $\frac{1}{\sqrt{b}} = \frac{1}{\sqrt{a}} + \frac{1}{\sqrt{c}}$ **(2)** a, b, c are in A.P.

(3) $\frac{1}{\sqrt{a}} = \frac{1}{\sqrt{b}} + \frac{1}{\sqrt{c}}$ **(4)** $\sqrt{a}, \sqrt{b}, \sqrt{c}$ are in A.P.

6. The maximum volume (in cu.m) of the right circular cone having slant height 3 m is:

(1) $3\sqrt{2}\pi$ **(2)** $2\sqrt{3}\pi$ **(3)** $\frac{4}{3}\pi$ **(4)** 6π

7. Let $f : R \to R$ be a function defined as $f(x) = \begin{cases} 5, & \text{if } x \le 1 \\ a + bx, & \text{if } 1 < x < 3 \\ b + 5x, & \text{if } 3 \le x < 3 \\ 30, & \text{if } x \ge 5 \end{cases}$

Then, f is:

(1) continuous if $a = -5$ and $b = 10$ **(2)** continuous if $a = 0$ and $b = 5$

(3) not continuous for any value of a and b **(4)** continuous if $a = 5$ and $b = 5$

8. Axis of a parabola lies along x-axis. If its vertex and focus are at distances 2 and 4 respectively from the origin, on the positive x-axis then which of the following points does not lie on it?

(1) (8, 6) **(2)** (4, –4) **(3)** $\left(6, 4\sqrt{2}\right)$ **(4)** $\left(5, 2\sqrt{6}\right)$

9. The plane through the intersection of the planes $x + y + z = 1$ and $2x + 3y - z + 4 = 0$ and parallel to y-axis also passes through the point:

(1) (3, 2, 1) **(2)** (3, 3, –1) **(3)** (–3, 1, 1) **(4)** (–3, 0, –1)

10. If the Boolean expression $(p \oplus q) \wedge (\sim p \odot q)$ is equivalent to $p \wedge q$, where $\oplus, \odot \in \{\wedge, \vee\}$, then the ordered pair $(\oplus, \odot)$ is:

(1) $(\wedge, \wedge)$ **(2)** $(\wedge, \vee)$ **(3)** $(\vee, \wedge)$ **(4)** $(\vee, \vee)$

11. The equation of the line passing through $(-4, 3, 1)$, parallel to the plane $x+2y-z-5=0$ and intersecting the line $\frac{x+1}{-3}=\frac{y-3}{2}=\frac{z-2}{-1}$ is:

(1) $\frac{x+4}{1}=\frac{y-3}{1}=\frac{z-1}{3}$ **(2)** $\frac{x+4}{3}=\frac{y-3}{-1}=\frac{z-1}{1}$

(3) $\frac{x-4}{2}=\frac{y+3}{1}=\frac{z+1}{4}$ **(4)** $\frac{x+4}{-1}=\frac{y-3}{1}=\frac{z-1}{1}$

12. Two cards are drawn successively with replacement from a well-shuffled deck of 52 cards. Let X denote the random variable of number of aces obtained in the two drawn cards. Then $P(X=1)+P(X=2)$ equals:

(1) $\frac{24}{169}$ **(2)** $\frac{52}{169}$ **(3)** $\frac{49}{169}$ **(4)** $\frac{25}{169}$

13. If $\cos^{-1}\left(\frac{2}{3x}\right)+\cos^{-1}\left(\frac{3}{4x}\right)=\frac{\pi}{2}\left(x>\frac{3}{4}\right)$, then x is equal to:

(1) $\frac{\sqrt{145}}{11}$ **(2)** $\frac{\sqrt{145}}{10}$ **(3)** $\frac{\sqrt{146}}{12}$ **(4)** $\frac{\sqrt{145}}{12}$

14. Consider a class of 5 girls and 7 boys. The number of different teams consisting of 2 girls and 3 boys that can be formed from this class, if there are two specific boys A and B, who refuse to be the members of the same team, is:

(1) 350 **(2)** 300 **(3)** 200 **(4)** 500

15. If the fractional part of the number $\frac{2^{403}}{15}$ is $\frac{k}{15}$, then k is equal to:

(1) 4 **(2)** 8 **(3)** 14 **(4)** 6

16. Equation of a common tangent to the circle, $x^2+y^2-6x=0$ and the parabola, $y^2=4x$ is:

(1) $2\sqrt{3}y=12x+1$ **(2)** $\sqrt{3}y=x+3$

(3) $2\sqrt{3}y=-x-12$ **(4)** $\sqrt{3}y=3x+1$

17. For any $\theta\in\left(\frac{\pi}{4},\frac{\pi}{2}\right)$, the expression $3(\sin\theta-\cos\theta)^4+6(\sin\theta+\cos\theta)^2+4\sin^6\theta$ equals:

(1) $13-4\cos^2\theta+6\sin^2\theta\cos^2\theta$ **(2)** $13-4\cos^4\theta+2\sin^2\theta\cos^2\theta$

(3) $13-4\cos^6\theta$ **(4)** $13-4\cos^2\theta+6\cos^4\theta$

18. If $A=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$, then the matrix A^{-50} when $\theta=\frac{\pi}{12}$, is equal to:

(1) $\begin{bmatrix}\frac{1}{2} & \frac{\sqrt{3}}{2}\\ -\frac{\sqrt{3}}{2} & \frac{1}{2}\end{bmatrix}$ **(2)** $\begin{bmatrix}\frac{\sqrt{3}}{2} & -\frac{1}{2}\\ \frac{1}{2} & \frac{\sqrt{3}}{2}\end{bmatrix}$ **(3)** $\begin{bmatrix}\frac{1}{2} & \frac{\sqrt{3}}{2}\\ \frac{1}{2} & \frac{1}{2}\end{bmatrix}$ **(4)** $\begin{bmatrix}\frac{\sqrt{3}}{2} & \frac{1}{2}\\ -\frac{1}{2} & \frac{\sqrt{3}}{2}\end{bmatrix}$

19. 5 students of a class have an average height 150 cm and variance 18 cm^2. A new student, whose height is 156 cm, joined them. The variance (in cm^2) of the height of these six students is:

(1) 22 **(2)** 20 **(3)** 18 **(4)** 16

20. Consider the set of all lines $px+qy+r=0$ such that $3p+2q+4r=0$. Which one of the following statements is true?

(1) The lines are not concurrent. **(2)** The lines are concurrent at the point $\left(\frac{3}{4}, \frac{1}{2}\right)$.

(3) The lines are all parallel. **(4)** Each line passes through the origin.

21. The area (in sq. Units) bounded by the parabola $y=x^2-1$, the tangent at the point (2, 3) to it and the y-axis is:

(1) $\frac{32}{3}$ **(2)** $\frac{56}{3}$ **(3)** $\frac{14}{3}$ **(4)** $\frac{8}{3}$

22. Let $A=\left\{\left\{\theta\in\left(-\frac{\pi}{2}, \pi\right): \frac{3+2i\sin\theta}{1-2i\sin\theta} \text{ is purely imaginary}\right\}\right.$. Then the sum of the elements in A is:

(1) π **(2)** $\frac{3\pi}{4}$ **(3)** $\frac{2\pi}{3}$ **(4)** $\frac{5\pi}{6}$

23. Let $0<\theta<\frac{\pi}{2}$. If the eccentricity of the hyperbola $\frac{x^2}{\cos^2\theta}-\frac{y^2}{\sin^2\theta}=1$ is greater than 2, then the length of its latus rectum lies in the interval:

(1) $(3, \infty)$ **(2)** $\left(\frac{3}{2}, 2\right]$ **(3)** $\left(1, \frac{3}{2}\right]$ **(4)** $(2, 3]$

24. Let $\vec{a}=\hat{i}-\hat{j}, \vec{b}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{c}$ be a vector such tht $\vec{a}\times\vec{c}+\vec{b}=\vec{0}$ and $\vec{a}\,.\,\vec{c}=4$, then $|\vec{c}|^2$ is equal to:

(1) 9 **(2)** 8 **(3)** $\frac{17}{2}$ **(4)** $\frac{19}{2}$

25. For $x\in R-\{0,1\}$, let $f_1(x)=\frac{1}{x}$, $f_2(x)=1-x$ and $f_3(x)=\frac{1}{1-x}$ be three given functions. If a function, $J(x)$ satisfies $(f_2.J.f_1)(x)=f_3(x)$ then $J(x)$ is equal to:

(1) $f_2(x)$ **(2)** $\frac{1}{x}f_3(x)$ **(3)** $f_1(x)$ **(4)** $f_3(x)$

26. $\lim\limits_{y\to0}\frac{\sqrt{1+\sqrt{1+y^4}}-\sqrt{2}}{y^4}$

(1) does not exist **(2)** exists and equals $\frac{1}{2\sqrt{2}}$

(3) exists and equals $\frac{1}{4\sqrt{2}}$ **(4)** exists and equals $\frac{1}{2\sqrt{2}(\sqrt{2}+1)}$

27. Let $a_1, a_2, \ldots\ldots, a_{30}$ be an A.P., $S=\sum\limits_{i=1}^{30}a_i$ and $T=\sum\limits_{i=1}^{15}a_{(2i-1)}$. If $a_5=27$ and $S-2T=75$, then a_{10} is equal to:

(1) 47 **(2)** 57 **(3)** 52 **(4)** 42

28. The system of linear equations

$x+y+z=2$

$2x+3y+2z=5$

$2x+3y+(a^2-1)z=a+1$

(1) is inconsistent when $|a|=\sqrt{3}$ **(2)** has infinitely many solutions for $a=4$

(3) is inconsistent when $a=4$ **(4)** has a unique solution for $|a|=\sqrt{3}$

29. The value of $\int_0^{\pi}|\cos x|^3\,dx$ is:

(1) 0 **(2)** $-\frac{4}{3}$ **(3)** $\frac{4}{3}$ **(4)** $\frac{2}{3}$

30. For $x^2 \neq n\pi+1,\ n\in N$ (the set of natural numbers), the integral

$\int x\sqrt{\frac{2\sin(x^2-1)-\sin 2(x^2-1)}{2\sin(x^2-1)+\sin 2(x^2-1)}}\,dx$ is:

(1) $\log_e\left|\sec\left(\frac{x^2-1}{2}\right)\right|+c$ **(2)** $\frac{1}{2}\log_e\left|\sec(x^2-1)\right|+c$

(3) $\frac{1}{2}\log_e\left|\sec^2\left(\frac{x^2-1}{2}\right)\right|+c$ **(4)** $\log_e\left|\frac{1}{2}\sec^2(x^2-1)\right|+c$