Joint Entrance Exam | Mains-2019

8^{th} April 2019 | Evening

PHYSICS , CHMISTRY & MATHEMATICS

Important Instructions:

1. Immediately fill in the particulars on this page of the Test Booklet with only Black Ball Point Pen provided in the examination hall.

2. The Answer Sheet is kept inside this Test Booklet. When you are directed to open the Test Booklet, take out the Answer Sheet and fill in the particulars carefully.

3. The test is of **3 hours** duration.

4. The Test Booklet consists of **90** questions. The maximum marks are **360**.

5. There are three parts in the question paper A, B, C consisting of **Physics, Mathematics** and **Chemistry** having 30 questions in each part of equal weightage. Each question is allotted **4 (four)** marks for correct response.

6. Candidate will be awarded marks as stated above in instruction No. 5 for correct response of each question. $\frac{1}{4}$ (one-fourth) marks of the total marks allotted to the questions (i.e. 1 mark) will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.

7. There is only one correct response for each question. Filling up more than one response in any question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 6 above.

8. For writing particulars/marking responses on *Side-1* and *Side-2* of the Answer Sheet use ***only Black Ball Point Pen*** provided in the examination hall.

9. No candidate is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, any electronic device, etc. except the Admit Card inside the examination room/hall.

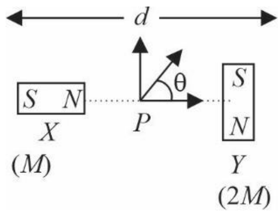
10. Rough work is to be done on the space provided for this purpose in the Test Booklet only. This space is given at the bottom of each page and in **four** pages (Page **20-23**) at the end of the booklet.

11. On completion of the test, the candidate must hand over the Answer Sheet to the **Invigilator** on duty in the Room/Hall. ***However, the candidates are allowed to take away this Test Booklet with them***.

12. The CODE for this Booklet is **B.** Make sure that the CODE printed on Side-2 of the Answer Sheet is same as that on this Booklet. Also tally the serial number of the Test Booklet and Answer Sheet are the same as that on this booklet. In case of discrepancy, the candidate should immediately report the matter to the Invigilator for replacement of both the Test Booklet and the Answer Sheet.

13. **Do not fold or make any stray mark on the Answer Sheet.**

1. Two magnetic dipoles X and Y are placed at a separation d, with their axes perpendicular to each other. The dipole moment of Y is twice that of X. A particle of charge q is passing through their midpoint P, at angle $\theta = 45°$ with the horizontal line, as shown in figure. What would be the magnitude of force on the particle at that instant? (d is much larger that the dimensions of the dipole)



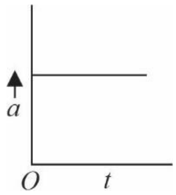
(1) $\left(\frac{\mu_0}{4\pi}\right)\frac{M}{\left(d/2\right)^3}\times qv$ **(2)** $\left(\frac{\mu_0}{4\pi}\right)\frac{2M}{\left(d/2\right)^3}\times qv$

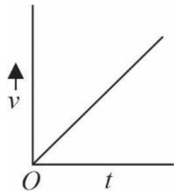
(3) $\sqrt{2}\left(\frac{\mu_0}{4\pi}\right)\frac{M}{\left(d/2\right)^3}\times qv$ **(4)** 0

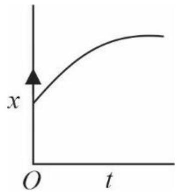
2. A parallel plate capacitor has $1\mu F$ capacitance. One of its two plates in given $+2\mu C$ charge and the other plate, $+4\mu C$ charge. The potential difference developed across the capacitor is:

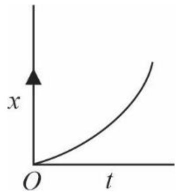
(1) $2V$ **(2)** $5V$ **(3)** $3V$ **(4)** $1V$

3. A particle starts form origin O from rest and moves with a uniform acceleration along the positive x-axis. Identify all figures that correctly represent the motion qualitatively. (a = acceleration, v = velocity, x = displacement, t = time)

(A) 

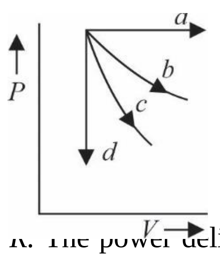
(B) 

(C) 

(D) 

(1) (B), (C) **(2)** (A) **(3)** (A), (B), (C) **(4)** (A), (B), (D)

4. The given diagram shows four processes i.e., isochoric, isobaric, isothermal and adiabatic. The correct assignment of the processes, in the same order is given by :



(1) $d\ a\ c\ b$ **(2)** $a\ d\ c\ b$

(3) $a\ d\ b\ c$ **(4)** $d\ a\ b\ c$

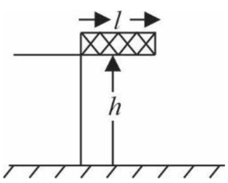
5. A cell of internal resistance r drives current through an external resistance R. The power delivered by the cell to the external resistance will be maximum when:

(1) $R = 0.001r$ **(2)** $R = 1000r$ **(3)** $R = 2r$ **(4)** $R = r$

6. A body of mass m_1 moving with an unknown velocity of $v_1\hat{i}$, undergoes a collinear collision with a body of mass m_2 moving with a velocity $v_2\hat{i}$. After collision, m_1 and m_2 move with velocities of $v_3\hat{i}$ and $v_4\hat{i}$, respectively. If $m_2 = 0.5m_1$ and $v_3 = 0.5v_1$, then v_1 is:

(1) $v_4 - \frac{v_2}{2}$ **(2)** $v_4 - v_2$ **(3)** $v_4 - \frac{v_2}{4}$ **(4)** $v_4 + v_2$

7. A rectangular solid box of length 0.3 m is held horizontally, with one of its sides on the edge of a platform of height $5m$. When released, it slips off the table in a very short time $\tau = 0.01s$, remaining essentially horizontally. The angle by which it would rotate when it hits the ground will be (in radians) close to:



(1) 0.3 **(2)** 0.28 **(3)** 0.02 **(4)** 0.5

8. Calculate the limit of resolution of a telescope objective having a diameter of 200 cm, if it has to detect light of wavelength 500 nm coming from a star.

(1)	305×10^{-9} radian	**(2)**	457.5×10^{-9} radian
(3)	152.5×10^{-9} radian	**(4)**	610×10^{-9} radian

9. A solid sphere and solid cylinder of identical radii approach an incline with the same linear velocity (see figure). Both roll without slipping all throughout. The two climb maximum heights h_{sph} and h_{cyl} on the incline. The ratio $\dfrac{h_{sph}}{h_{cyl}}$ is given by

(1) 1 **(2)** $\dfrac{2}{\sqrt{5}}$ **(3)** $\dfrac{14}{15}$ **(4)** $\dfrac{4}{5}$

10. The magnetic field of an electromagnetic wave is given by:

$$\vec{B}=1.6\times10^{-6}\cos\left(2\times10^{7}z+6\times10^{15}t\right)\left(2\hat{i}+\hat{j}\right)\frac{Wb}{m^2}$$

The associated electric field will be:

(1) $\vec{E}=4.8\times10^{2}\cos\left(2\times10^{7}z+6\times10^{15}t\right)\left(\hat{i}-2\hat{j}\right)\dfrac{V}{m}$

(2) $\vec{E}=4.8\times10^{2}\cos\left(2\times10^{7}z+6\times10^{15}t\right)\left(-\hat{i}+2\hat{j}\right)\dfrac{V}{m}$

(3) $\vec{E}=4.8\times10^{2}\cos\left(2\times10^{7}z-6\times10^{15}t\right)\left(-2\hat{j}+\hat{i}\right)\dfrac{V}{m}$

(4) $\vec{E}=4.8\times10^{2}\cos\left(2\times10^{7}z-6\times10^{15}t\right)\left(2\hat{i}+\hat{j}\right)\dfrac{V}{m}$

11. If Surface tension (S), Moment of Inertia (I) and Planck's constant (h), were to be taken as the fundamental units, the dimensional formula for linear momentum would be :

(1) $S^{1/2}I^{3/2}h^{-1}$ **(2)** $S^{1/2}I^{1/2}h^{-1}$ **(3)** $S^{1/2}I^{1/2}h^{0}$ **(4)** $S^{3/2}I^{1/2}h^{0}$

12. A rocket has to be launched from earth in such a way that it never returns. If E is the minimum energy delivered by the rocket launcher, what should be the minimum energy that the launcher should have if the same rocket is to be launched from the surface of the moon? Assume that the density of the earth and the moon are equal and that the earth's volume is 64 times the volume of the moon.

(1) $\dfrac{E}{64}$ **(2)** $\dfrac{E}{32}$ **(3)** $\dfrac{E}{16}$ **(4)** $\dfrac{E}{4}$

13. Let $\left|\vec{A_1}\right|=3,\left|\vec{A_2}\right|=5$ and $\left|\vec{A_1}+\vec{A_2}\right|=5$. The value of $\left(2\vec{A_1}+3\vec{A_2}\right).\left(3\vec{A_1}+2\vec{A_2}\right)$ is:

(1) -118.5 **(2)** -106.5 **(3)** -112.5 **(4)** -99.5

14. The ratio of mass densities of nuclei of ^{40}Ca and ^{16}O is close to:

(1) 1 **(2)** 0.1 **(3)** 2 **(4)** 5

15. Young's moduli of two wires A and B are in the ratio $7:4$. Wire A is 2 m long and has radius R. Wire B is 1.5 m long and has radius 2 mm. If the two wires stretch by the same length for a given load, then the value of R is close to:

(1) $1.5mm$ **(2)** $1.3mm$ **(3)** $1.7mm$ **(4)** $1.9mm$

16. A circuit connected to an ac source of emf $e = e_0 \sin(100t)$ with t in seconds, gives a phase difference of $\frac{\pi}{4}$ between the emf e and current i. Which of the following circuits will exhibits this?

(1) RC circuit with $R = 1\,k\Omega$ and $C = 10\mu F$

(2) RL circuit with $R = 1\ k\Omega$ and $L = 1\,mH$

(3) RL circuit with $R = 1\ k\Omega$ and $L = 10mH$

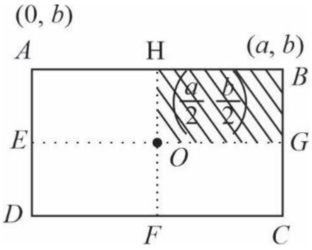
(4) RC circuit with $R = 1\,k\Omega$ and $C = 1\mu F$

17. The temperature, at which the root mean square velocity of hydrogen molecules equals their escape velocity from the earth, is closest to:

[Boltzmann Constant $k_B = 1.38 \times 10^{-23} J/K$ Avogadro Number $N_A = 6.02 \times 10^{26}/kg$ Radius of Earth : $6.4 \times 10^6 m$ Gravitational acceleration on Earth = $10ms^{-2}$]

(1) $10^4 K$ **(2)** $800K$ **(3)** $650K$ **(4)** $3 \times 10^5 K$

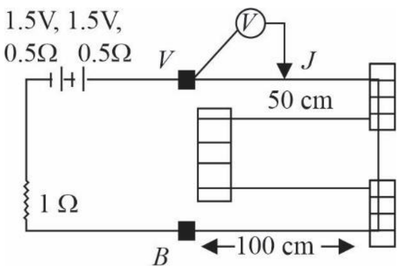
18. A uniform rectangular thin sheet $ABCD$ of mass M has length a and breadth b, as shown in the figure. If the shaded portion $HBGO$ is cut-off, the coordinates of the centere of mass of the remaining portion will be :



(1) $\left(\frac{5a}{3}, \frac{5b}{3}\right)$ **(2)** $\left(\frac{5a}{12}, \frac{5b}{12}\right)$

(3) $\left(\frac{2a}{3}, \frac{2b}{3}\right)$ **(4)** $\left(\frac{3a}{4}, \frac{3b}{4}\right)$

19. In the circuit shown, a four-wire potentiometer is made of a 400 cm long wire, which extends between A and B. The resistance per unit length of the potentiometer wire is $r = 0.01\Omega/cm$. If an ideal voltmeter is connected as shown with jockey J at 50 cm from end A, the expected reading of the voltmeter will be :



(1) $0.25V$ **(2)** $0.75V$

(3) $0.50V$ **(4)** $0.20V$

20. In a simple pendulum experiment for determination of acceleration due to gravity (g), time taken for 20 oscillation is measured by using a watch of 1 second least count. The mean value of time taken comes out to be 30 s. The length of pendulum is measured by using a meter scale of least count 1 mm and the value obtained is 55.0 cm. The percentage error in the determination of g is close to :

(1) 0.7% **(2)** 6.8% **(3)** 3.5% **(4)** 0.2%

21. A damped harmonic oscillator has a frequency of 5 oscillations per second. The amplitude drops to half its value for every 10 oscillations. The time it will take to drop to $\frac{1}{1000}$ of the original amplitude is close to:

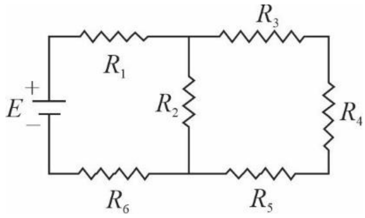
(1) $100s$ **(2)** $10s$ **(3)** $10s$ **(4)** $20s$

22. A convex lens (of focal length 20 cm) and a concave mirror, having their principal axes along the same lines, are kept 80 cm apart from each other. The concave mirror is to the right of the convex lens. When an object is kept at a distance of 30 cm to the left of the convex lens, its image remains at the same position even if the concave mirror is removed. The maximum distance of the object for which this concave mirror, by itself would product a virtual image would be:

(1) $10cm$ **(2)** $20cm$ **(3)** $20\ cm$ **(4)** $30cm$

23. In the figure shown, what is the current (in Ampere) drawn from the battery? You are given:

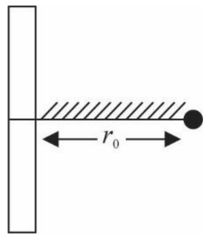
$R_1 = 15\Omega,\ R_2 = 10\Omega, R_3 = 20\Omega, R_4 = 5\Omega,\ \ R_5 = 25\Omega, R_6 = 30\Omega, E = 15V$



(1) $13/24$ **(2)** $20/3$

(3) $7/18$ **(4)** $9/32$

24. A positive point charge is released from rest at a distance r_0 from a positive line charge with uniform density. The speed (v) of the point charge, as a function of instantaneous distance r from line charge, is proportional to:



(1) $v \propto e^{+r/r_0}$ **(2)** $v \propto \ln\left(\frac{r}{r_0}\right)$

(3) $v \propto \left(\frac{r}{r_0}\right)$ **(4)** $v \propto \sqrt{\ln\left(\frac{r}{r_0}\right)}$

25. The electric field in a region is given by $\vec{E} = (Ax + B)\hat{i}$, where E is in NC^{-1} and x is in metres. The values of constants are $A = 20\ SI$ unit and $B = 10\ SI$ unit. If the potential at $x = 1$ is V_1 and that at $x = -5$ is V_2, then $V_1 - V_2$ is :

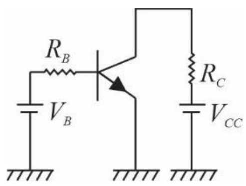
(1) $-520V$ **(2)** $320V$ **(3)** $-48V$ **(4)** $180V$

26. In a line of sight radio communication, a distance of about 50 km is kept between the transmitting and receiving antennas. If the height of the receiving antenna is 70 m, then the minimum height of the transmitting antenna should be:

(Radius of the Earth $= 6.4 \times 10^6 m$).

(1) $51\ m$ **(2)** $40 m$ **(3)** $20\ m$ **(4)** $32\ m$

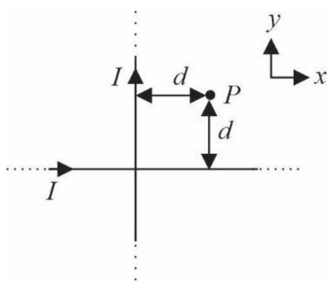
27. A common emitter amplifier circuit, built using an npn transistor, is shown in the figure. Its dc current gain is 250, $R_C = 1k\Omega$ and $V_{CC} = 10V$. What is the minimum base current for V_{CE} to reach saturation?



(1) $40\ \mu A$ **(2)** $7\ \mu A$

(3) $100\ \mu A$ **(4)** $10\ \mu A$

28. Two very long, straight, and insulated wires are kept at 90° angle from each other in xy-plane as shown in the figure. These wires carry currents of equal magnitude I, whose directions are shown in the figure. The net magnetic field at point P will be:



(1) $\frac{+\mu_0 I}{\pi d}(\hat{z})$ **(2)** Zero

(3) $\frac{\mu_0 I}{2\pi d}(\hat{x} + \hat{y})$ **(4)** $-\frac{\mu_0 I}{2\pi d}(\hat{x} + \hat{y})$

29. An electric dipole is formed by two equal and opposite charges q with separation d. The charges have same mass m. It is kept in a uniform electric field E. If it is slightly rotated from its equilibrium orientation, then its angular frequency ω is:

(1) $\sqrt{\frac{qE}{2md}}$ **(2)** $\sqrt{\frac{qE}{md}}$ **(3)** $\sqrt{\frac{2qE}{md}}$ **(4)** $2\sqrt{\frac{qE}{md}}$

30. A nucleus A, with a finite de-broglie wavelength λ_A, undergoes spontaneous fission in to two nuclei B and C of equal mass. B flies in the same direction as that of A, while C flies in the opposite direction with a velocity equal to half of that of B. The de-Broglie wavelengths λ_B and λ_C of B and C are respectively:

(1) $\frac{\lambda_A}{2}, \lambda_A$ **(2)** $\lambda_A, 2\lambda_A$ **(3)** $2\lambda_A, \lambda_A$ **(4)** $\lambda_A, \frac{\lambda_A}{2}$

PART-B	CHEMISTRY

1. The correct statement about ICl_5 and ICl_4^- is:

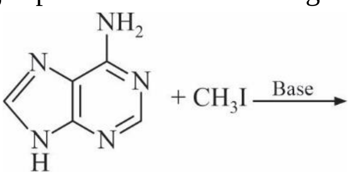
(1) ICl_5 is trigonal bipyramidal and ICl_4^- is tetrahedral

(2) ICl_5 is square pyramidal and ICl_4^- is square planar.

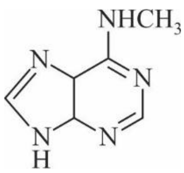
(3) both are isostructural

(4) ICl_5 is square pyramidal and ICl_4^- is tetrahedral.

2. The major product in the following reaction is:



(1)



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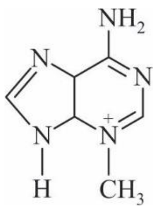

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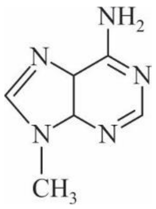

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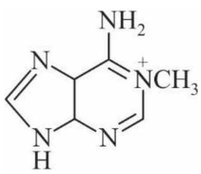


3. The percentage composition of carbon by mole in methane is:

(1) 25% **(2)** 80% **(3)** 75% **(4)** 20%

4. Among the following molecules/ions, $C_2^{2-}, N_2^{2-}, O_2^{2-}, O_2$

Which one is diamagnetic and has the shortest bond length?

(1) C_2^{2-} **(2)** O_2^{2-} **(3)** O_2 **(4)** N_2^{2-}

5. 5 moles of an ideal gas at 100 K are allowed to undergo reversible compression till its temperature becomes 200 K. If $C_V = 28 J\ K^{-1} mol^{-1}$, calculate ΔU and ΔpV for this process. $\left(R = 8.0 J\ K^{-1}\ mol^{-1}\right)$

(1) $\Delta U = 14 kJ; \Delta(pV) = 4 kJ$

(2) $\Delta U = 14 J; \Delta(pV) = 0.8 J$

(3) $\Delta U = 14 kJ; \Delta(pV) = 18 kJ$

(4) $\Delta U = 28 kJ; \Delta(pV) = 0.8 kJ$

6. The statement that is INCORRECT about the interstitial compounds is:

(1) they are very hard

(2) they have metallic conductivity

(3) they are chemically reactive

(4) they have high melting points

7. The Mond process is used for the:

(1) extraction of Zn

(2) purification of Zr and Ti

(3) purification of Ni

(4) extraction of Mo

8. For a reaction scheme $A \xrightarrow{k_1} B \xrightarrow{k_2} C$, if the rate of formation of B is set to be zero then the concentration of B is given by:

(1) $(k_1+k_2)[A]$ **(2)** $(k_1-k_2)[A]$ **(3)** $k_1k_2[A]$ **(4)** $\left(\frac{k_1}{k_2}\right)[A]$

9. Which one of the following alkenes when treated with HCl yields majorly an anti Markovnikov product?

(1) $F_3C-CH=CH_2$ **(2)** $Cl-CH=CH_2$

(3) $H_2N-CH=CH_2$ **(4)** $CH_3O-CH=CH_2$

10. The covalent alkaline earth metal halide $(X=Cl, Br, I)$ is:

(1) MgX_2 **(2)** CaX_2 **(3)** SrX_2 **(4)** BeX_2

11. The compound that inhibits the growth of tumors is:

(1) trans-$[Pd(Cl)_2(NH_3)_2]$ **(2)** cis$-[Pt(Cl)_2(NH_3)_2]$

(3) trans$-[Pt(Cl)_2(NH_3)_2]$ **(4)** cis$-[Pd(Cl)_2(NH_3)_2]$

12. For the following reactions, equilibrium constants are given:

$S(s)+O_2(g) \rightleftharpoons SO_2(g); K_1=10^{52}$

$2S(s)+3O_2(g) \rightleftharpoons 2SO_3(g); K_2=10^{129}$

The equilibrium constant for the reaction, $2SO_2(g)+O_2(g) \rightleftharpoons 2SO_3(g)$ is:

(1) 10^{77} **(2)** 10^{154} **(3)** 10^{25} **(4)** 10^{181}

13. Fructose and glucose can be distinguished by:

(1) Seliwanoff's test **(2)** Benedict's test

(3) Barfoed's test **(4)** Fehling's test

14. The IUPAC symbol for the element with atomic number 119 would be:

(1) unh **(2)** uun **(3)** uue **(4)** une

15. 0.27 g of a long chain fatty acid was dissolved in $100\,cm^3$ of hexane. 10 mL of this solution was added dropwise to the surface of water in a round watch glass. Hexane evaporates and a monolayer is formed. The distance from edge to centre of the watch glass is 10 cm. What is the height of the monolayer?

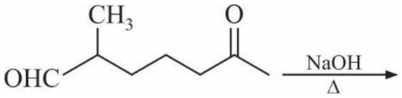
[Density of fatty acid $=0.9g\ cm^{-3}, \pi=3$**]**

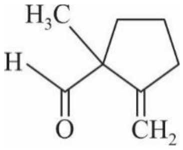
(1) $10^{-4}m$ **(2)** $10^{-2}m$ **(3)** $10^{-6}m$ **(4)** $10^{-8}m$

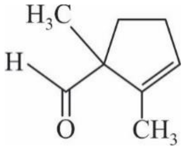
16. The ion that has sp^3d^2 hybridization for the central atom, is:

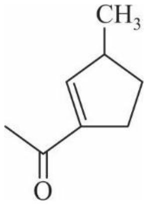
(1) $[ICl_4]^-$ **(2)** $[ICl_2]^-$ **(3)** $[BrF_2]^-$ **(4)** $[IF_6]^-$

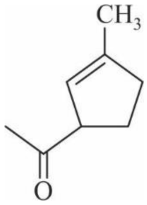
17. The major product obtained in the following reaction is :



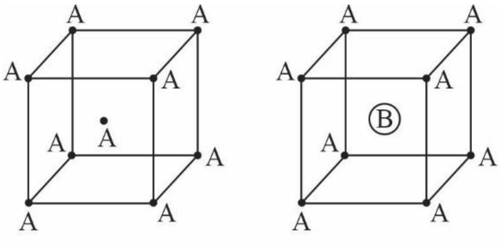
(1) 

(2) 

(3) 

(4) 

18. Consider the bcc unit cells of the solids 1 and 2 with the position of atoms as shown below. The radius of atom B is twice that of atom A. The unit cell edge length is 50% more in solid 2 than in 1. What is the approximate packing efficiency in solid 2?



Solid 1 **Solid 2**

(1) 65% **(2)** 45% **(3)** 90% **(4)** 75%

19. The structure of Nylon-6 is:

(1) $\left[\overset{\overset{\large O}{\|}}{C} - (CH_2)_5 - \overset{\overset{\large H}{|}}{N} \right]_n$

(2) $\left[(CH_2)_6 - \overset{\overset{\large O}{\|}}{C} - \overset{\overset{\large H}{|}}{N} \right]_n$

(3) $\left[(CH_2)_4 - \overset{\overset{\large O}{\|}}{C} - \overset{\overset{\large H}{|}}{N} \right]_n$

(4) $\left[\overset{\overset{\large O}{\|}}{C} - (CH_2)_6 - \overset{\overset{\large H}{|}}{N} \right]_n$

20. Polysubstitution is a major drawback in:

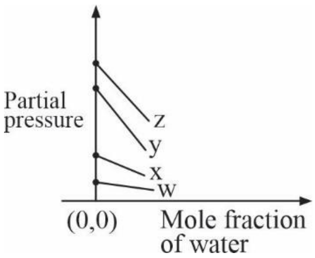
(1) Reimer tiemann reaction **(2)** Friedel Craft's alkylation

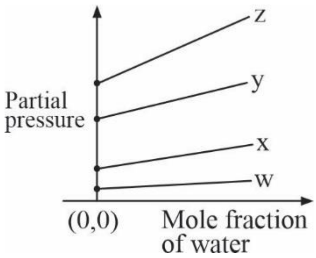
(3) Acetylation of aniline **(4)** Friedel Craft's acylation

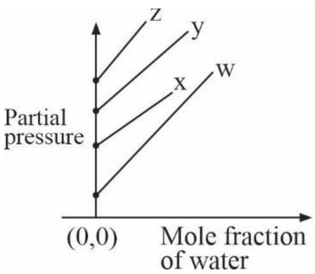
21. The strength of 11.2 volume solution of H_2O_2 is: [Given that molar mass of $H = 1g\ mol^{-1}$ and $O = 16g\ mol^{-1}$]

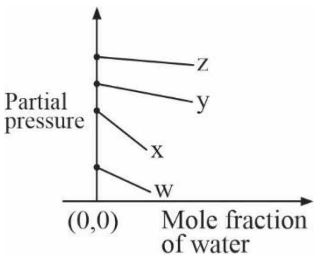
(1) 3.4% **(2)** 13.6% **(3)** 34% **(4)** 1.7%

22. For the solution of the gases w, x, y and z in water at 298 K, the Henrys law constants (K_H) are $0.5, 2, 35$ and 40 kbar, respectively. The correct plot for the given data is :

(1) 

(2) 

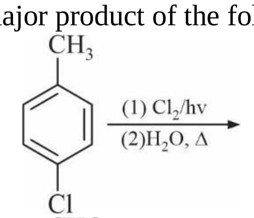
(3) 

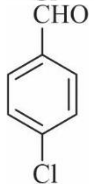
(4) 

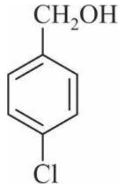
23. The calculated spin-only magnetic moments (BM) of the anionic and cationic species of $[Fe(H_2O)_6]_2$ and $[Fe(CN)_6]$, respectively, are:

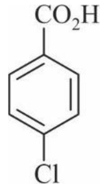
(1) 4.9 and 0 **(2)** 0 and 4.9 **(3)** 0 and 5.92 **(4)** 2.84 and 5.92

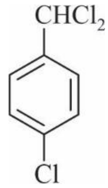
24. The major product of the following reaction is:



(1) 

(2) 

(3) 

(4) 

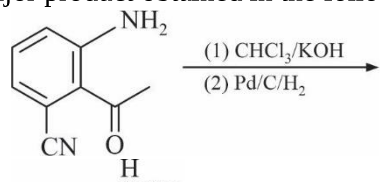
25. The maximum prescribed concentration of copper in drinking water is:

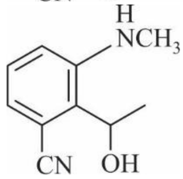
(1) 0.5 ppm **(2)** 5 ppm **(3)** 0.05 ppm **(4)** 3 ppm

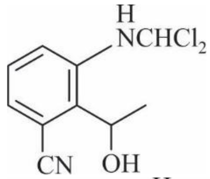
26. If p is the momentum of the fastest electron ejected from a metal surface after the irradiation of light having wavelength λ, then for $1.5\,p$ momentum of the photoelectron, the wavelength of the light should be: (Assume kinetic energy of ejected photoelectron to be very high in comparison to work function):

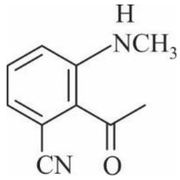
(1) $\frac{4}{9}\lambda$ **(2)** $\frac{3}{4}\lambda$ **(3)** $\frac{1}{2}\lambda$ **(4)** $\frac{2}{3}\lambda$

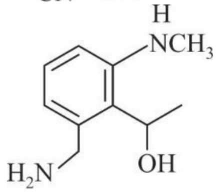
27. The major product obtained in the following reaction is:



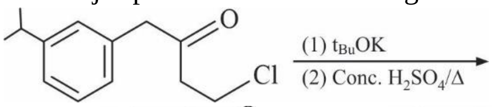
(1) 

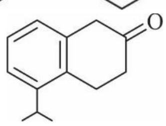
(2) 

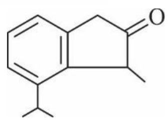
(3) 

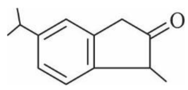
(4) 

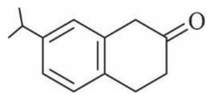
28. The major product of the following reaction is:



(1) 

(2) 

(3) 

(4) 

29. Which of the following compounds will show the maximum 'enol' content?

(1) $CH_3COCH_2COCH_3$ **(2)** $CH_3COCH_2CONH_2$

(3) $CH_3COCH_2COOC_2H_5$ **(4)** CH_3COCH_3

30. Calculate the standard cell potential (in V) of the cell in which following reaction takes place:

$$Fe^{2+}(aq) + Ag^{+}(aq) \rightarrow Fe^{3+}(aq) + Ag(s)$$

Given that

$E^0_{Ag^+/Ag} = xV$; $E^0_{Fe^{2+}/Fe} = yV$; $E^0_{Fe^{3+}/Fe} = zV$

(1) $x - z$ **(2)** $x + y - z$ **(3)** $x + 2y - 3z$ **(4)** $x - y$

PART-C	MATHEMATICS

1. The sum $\sum_{k=1}^{20} k\frac{1}{2^k}$ is equal to:

(1) $2-\frac{21}{2^{20}}$ **(2)** $2-\frac{3}{2^{17}}$ **(3)** $2-\frac{11}{2^{19}}$ **(4)** $1-\frac{11}{2^{20}}$

2. The tangent and the normal lines at the point $\left(\sqrt{3},1\right)$ to the circle $x^2+y^2=4$ and the x-axis form a triangle . The area of this triangle (in square units) is:

(1) $\frac{1}{\sqrt{3}}$ **(2)** $\frac{1}{3}$ **(3)** $\frac{2}{\sqrt{3}}$ **(4)** $\frac{4}{\sqrt{3}}$

3. The number of four-digit numbers strictly greater than 4321 that can be formed using the digits 0,1,2,3,4,5 (repetition of digits is allowed) is:

(1) 306 **(2)** 310 **(3)** 288 **(4)** 360

4. Let $f:R\to R$ be a differentiable function satisfying $f'(3)+f'(2)=0.$ Then $\lim_{x\to 0}\left(\frac{1+f(3+x)-f(3)}{1+f(2-x)-f(2)}\right)^{\frac{1}{x}}$ is equal to:

(1) e^{-1} **(2)** e^2 **(3)** e **(4)** 1

5. Suppose that the points $(h,k),(1,2)$ and $(-3,4)$ lie on the line L_1. If a line L_2 passing through the points (h,k) and $(4,3)$ is perpendicular to L_1, then $\frac{k}{h}$ equals:

(1) 3 **(2)** $\frac{1}{3}$ **(3)** 0 **(4)** $-\frac{1}{7}$

6. Given that the slope of the tangent to a curve $y=y(x)$ at any point (x,y) is $\frac{2y}{x^2}$. If the curve passes through the centre of the circle $x^2+y^2-2x-2y=0,$ then its equation is:

(1) $x\log_e|y|=2(x-1)$ **(2)** $x^2\log_e|y|=-2(x-1)$

(3) $x\log_e|y|=x-1$ **(4)** $x\log_e|y|=-2(x-1)$

7. The tangent to the parabola $y^2=4x$ at the point where it intersects the circle $x^2+y^2=5$ in the first quadrant, passes through the point:

(1) $\left(-\frac{1}{3},\frac{4}{3}\right)$ **(2)** $\left(-\frac{1}{4},\frac{1}{2}\right)$ **(3)** $\left(\frac{1}{4},\frac{3}{4}\right)$ **(4)** $\left(\frac{3}{4},\frac{7}{4}\right)$

8. The number of integral values of m for which the equation $\left(1+m^2\right)x^2-2(1+3m)x+(1+8m)=0$ has no real root is :

(1) 3 **(2)** 1

(3) 2 **(4)** infinitely many

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9. If the fourth term in the binomial expansion of $\left(\sqrt{\frac{1}{x^{1+\log_{10} x}}}+x^{\frac{1}{12}}\right)^6$ is equal to 200, and $x>1$, then the value of x is:

(1) 10^4 **(2)** 100 **(3)** 10 **(4)** 10^3

10. If the system of linear equations
$x-2y+kz=1;\ 2x+y+z=2;\ 3x-y-kz=3$
has a solution $(x,y,z), z\neq 0$, then (x,y) lies on the straight line whose equation is:

(1) $3x-4y-1=0$ **(2)** $3x-4y-4=0$ **(3)** $4x-3y-4=0$ **(4)** $4x-3y-1=0$

11. The height of a right circular cylinder of maximum volume inscribed in a sphere of radius 3 is:

(1) $\sqrt{6}$ **(2)** $\sqrt{3}$ **(3)** $\frac{2}{3}\sqrt{3}$ **(4)** $2\sqrt{3}$

12. Let $f(x)=a^x\ (a>0)$ be written as $f(x)=f_1(x)+f_2(x)$, where $f_1(x)$ is an even function and $f_2(x)$ is an odd function. Then $f_1(x+y)+f_1(x-y)$ equals:

(1) $2f_1(x)f_2(x)$ **(2)** $2f_1(x+y)f_2(x-y)$

(3) $2f_1(x+y)f_1(x-y)$ **(4)** $2f_1(x)f_1(y)$

13. If the eccentricity of the standard hyperbola passing through the point (4, 6) is 2, then the equation of the tangent to the hyperbola at (4, 6) is:

(1) $x-2y+8=0$ **(2)** $2x-y-2=0$ **(3)** $3x-2y=0$ **(4)** $2x-3y+10=0$

14. If the lengths of the sides of a triangle are in A.P. and the greatest angle is double the smallest, then a ratio of lengths of the sides of this triangle is:

(1) $5:9:13$ **(2)** $3:4:5$ **(3)** $4:5:6$ **(4)** $5:6:7$

15. If a point $R(4,y,z)$ lies on the line segment joining the points $P(2,-3,4)$ and $Q(8,0,10)$, then the distance of R from the origin is:

(1) 6 **(2)** $\sqrt{53}$ **(3)** $2\sqrt{14}$ **(4)** $2\sqrt{21}$

16. If $\int \frac{dx}{x^3\left(1+x^6\right)^{2/3}}=xf(x)\left(1+x^6\right)^{\frac{1}{3}}+C$ where C is a constant of integration, then the function $f(x)$ is equal to:

(1) $\frac{3}{x^2}$ **(2)** $-\frac{1}{2x^3}$ **(3)** $-\frac{1}{2x^2}$ **(4)** $-\frac{1}{6x^3}$

17. If three distinct numbers a,b,c are G.P. and the equations $ax^2+2bx+c=0$ and $dx^2+2ex+f=0$ have a common root, then which one of the following statements is correct?

(1) $\frac{d}{a},\frac{e}{b},\frac{f}{c}$ are in G.P. **(2)** d,e,f are in G.P.

(3) d,e,f are in A.P. **(4)** $\frac{d}{a},\frac{e}{b},\frac{f}{c}$ are in A.P.

18. Which one of the following statements is not a tautology?

(1) $(p \vee q) \to (p \vee (\sim q))$ **(2)** $(p \wedge q) \to (\sim p) \vee q$

(3) $p \to (p \vee q)$ **(4)** $(p \wedge q) \to p$

19. Let $f(x) = \int_0^x g(t)dt$, where g is a non-zero even function. If $f(x+5) = g(x)$, then $\int_0^x f(t)dt$ equals:

(1) $5\int_{x+5}^{5} g(t)dt$ **(2)** $2\int_{5}^{x+5} g(t)dt$ **(3)** $\int_{5}^{x+5} g(t)dt$ **(4)** $\int_{x+5}^{5} g(t)dt$

20. Let $\vec{a} = 3\hat{i} + 2\hat{j} + x\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$, for some real x. Then $|\vec{a} \times \vec{b}| = r$ is possible if:

(1) $3\sqrt{\frac{3}{2}} < r < 5\sqrt{\frac{3}{2}}$ **(2)** $0 < r \le \sqrt{\frac{3}{2}}$

(3) $r \ge 5\sqrt{\frac{3}{2}}$ **(4)** $\sqrt{\frac{3}{2}} < r \le 3\sqrt{\frac{3}{2}}$

21. If $f(1) = 1, f'(1) = 3,$ then the derivative of $f(f(f(x))) + (f(x))^2$ at $x = 1$ is.

(1) 15 **(2)** 12 **(3)** 9 **(4)** 33

22. Two vertical poles of heights, 20 m and 80 m stand apart on a horizontal plane. The height (in meters) of the point of intersection of the lines joining the top of each pole to the foot of the other, form this horizontal plane is:

(1) 12 **(2)** 18 **(3)** 16 **(4)** 15

23. The minimum number of times one has to toss a fair coin so that the probability of observing at least one head is at least 90% is:

(1) 2 **(2)** 4 **(3)** 5 **(4)** 3

24. If $z = \frac{\sqrt{3}}{2} + \frac{i}{2}\left(i = \sqrt{-1}\right)$, then $\left(1 + iz + z^5 + iz^8\right)^9$ is equal to:

(1) $(-1+2i)^9$ **(2)** 0 **(3)** –1 **(4)** 1

25. A student scores the following marks in five tests: 45, 54, 41, 57, 43. His score is not known for the sixth test. If the mean score is 48 in the six tests, then the standard deviation of the marks in six tests is:

(1) $\frac{100}{3}$ **(2)** $\frac{10}{3}$ **(3)** $\frac{10}{\sqrt{3}}$ **(4)** $\frac{100}{\sqrt{3}}$

26. In an ellipse, with centre at the origin, if the difference of the lengths of major axis and minor axis is 10 and one of the foci is at $\left(0, 5\sqrt{3}\right)$, then the length of its latus rectum is:

(1) 6 **(2)** 8 **(3)** 5 **(4)** 10

27. Let $S(\alpha) = \left\{(x,y) : y^2 \le x, 0 \le x \le \alpha\right\}$ and $A(\alpha)$ is area of the region $S(\alpha)$. If for a $\lambda, 0 < \lambda < 4, A(\lambda) : A(4) = 2:5,$ then λ equals:

(1) $4\left(\frac{4}{25}\right)^{\frac{1}{3}}$ **(2)** $2\left(\frac{2}{5}\right)^{\frac{1}{3}}$ **(3)** $2\left(\frac{4}{25}\right)^{\frac{1}{3}}$ **(4)** $4\left(\frac{2}{5}\right)^{\frac{1}{3}}$

28. Let the numbers $2, b, c$ be in an A.P. and $A = \begin{vmatrix} 1 & 1 & 1 \\ 2 & b & c \\ 4 & b^2 & c^2 \end{vmatrix}$. If det $(A) \in [2, 16]$, then c lies in the interval.

(1) $[2, 3)$ **(2)** $\left[3, 2+2^{3/4}\right]$ **(3)** $[4, 6]$ **(4)** $(2+2^{3/4}, 4)$

29. The vector equation of the plane through the line of intersection of the planes $x+y+z=1$ and $2x+3y+4z=5$ which is perpendicular to the plane $x-y+z=0$ is :

(1) $\vec{r} \times (\hat{i}+\hat{k})+2=0$ **(2)** $\vec{r} \times (\hat{i}-\hat{k})+2=0$

(3) $\vec{r} \cdot (\hat{i}-\hat{k})-2=0$ **(4)** $\vec{r} \cdot (\hat{i}-\hat{k})+2=0$

30. Let $f : [-1, 3] \to R$ be defined as

$$f(x) = \begin{cases} |x| + [x], & -1 \le x < 1 \\ x + |x|, & 1 \le x < 2 \\ x + [x], & 2 \le x \le 3, \end{cases}$$

Where [t] denotes the greatest integer less than or equal to t. Then, f is discontinuous at:

(1) only three point **(2)** four or more points

(3) only two points **(4)** only one point