



KVPY QUESTION PAPER –STREAM SA

October 27, 2013

MATHEMATICS

- 1 Let x, y, z be three non-negative integers such that $x + y + z = 10$. The maximum possible value of $xyz + xy + yz + zx$ is
- A. 52 B. 64
C. 69 D. 73
- 2 If a, b are natural numbers such that $2013 + a^2 = b^2$, then the minimum possible value of ab is
- A. 671 B. 668
C. 658 D. 645
- 3 The number of values of b for which there is an isosceles triangle with sides of lengths $b + 5, 3b - 2$ and $6 - b$ is
- A. 0 B. 1
C. 2 D. 3
- 4 Let a, b be non-zero real numbers. Which of the following statements about the quadratic equation $ax^2 + (a + b)x + b = 0$ is necessarily true?
- (I) It has at least one negative root.
(II) It has at least one positive root.
(III) Both its roots are real.
- A. (I) and (II) only B. (I) and (III) only
C. (II) and (III) only D. All of them

- 5 Let x, y, z be non-zero real numbers such that $\frac{x}{y} + \frac{y}{z} + \frac{z}{x} = 7$ and $\frac{y}{x} + \frac{z}{y} + \frac{x}{z} = 9$, then $\frac{x^3}{y^3} + \frac{y^3}{z^3} + \frac{z^3}{x^3} - 3$ is equal to

- A. 152
B. 153
C. 154
D. 155

- 6 In a triangle ABC with $\angle A < \angle B < \angle C$, points D, E, F are on the interior of segments BC, CA, AB , respectively. Which of the following triangles CANNOT be similar to ABC ?

- A. Triangle ABD
B. Triangle BCE
C. Triangle CAF
D. Triangle DEF

- 7 Tangents to a circle at points P and Q on the circle intersect at a point R . If $PQ = 6$ and $PR = 5$ then the radius of the circle is

- A. $\frac{13}{3}$
B. 4
C. $\frac{15}{4}$
D. $\frac{16}{5}$

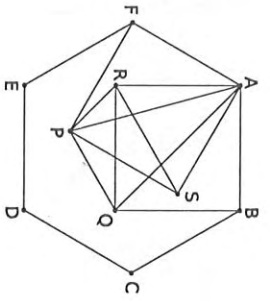
- 8 In an acute-angled triangle ABC , the altitudes from A, B, C when extended intersect the circumcircle again at points A_1, B_1, C_1 , respectively. If $\angle ABC = 45^\circ$ then $\angle A_1B_1C_1$ equals

- A. 45°
B. 60°
C. 90°
D. 135°

- 9 In a rectangle $ABCD$, points X and Y are the midpoints of AD and DC , respectively. Lines BX and CD when extended intersect at E , lines BY and AD when extended intersect at F . If the area of $ABCD$ is 60 then the area of BEF is

- A. 60
B. 80
C. 90
D. 120

- 10 In the figure given below, $ABCDEF$ is a regular hexagon of side length 1, $AFPS$ and $ABQR$ are squares. Then the ratio $\text{Area}(APQ)/\text{Area}(SRP)$ equals



- A. $\frac{\sqrt{2}+1}{2}$
B. $\sqrt{2}$
C. $\frac{3\sqrt{3}}{4}$
D. 2

- 11 A person X is running around a circular track completing one round every 40 seconds. Another person Y running in the opposite direction meets X every 15 seconds. The time, expressed in seconds, taken by Y to complete one round is

- A. 12.5
B. 24
C. 25
D. 55

12 The least positive integer n for which

$$\sqrt{n+1} - \sqrt{n-1} < 0.2 \text{ is}$$

- A. 24
- B. 25
- C. 26
- D. 27

13 How many natural numbers n are there such that $n!+10$ is a perfect square?

- A. 1
- B. 2
- C. 4
- D. infinitely many

14 Ten points lie in a plane so that no three of them are collinear. The number of lines passing through exactly two of these points and dividing the plane into two regions each containing four of the remaining points is

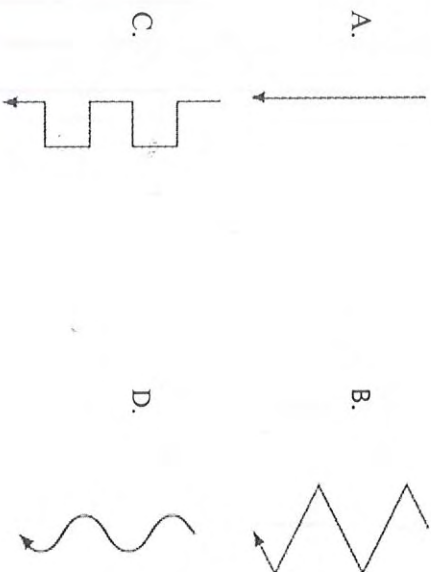
- A. 1
- B. 5
- C. 10
- D. dependent on the configuration of points

15 In a city, the total income of all people with salary below Rs. 10000 per annum is less than the total income of all people with salary above Rs. 10000 per annum. If the salaries of people in the first group increases by 5% and the salaries of people in the second group decreases by 5% then the average income of all people

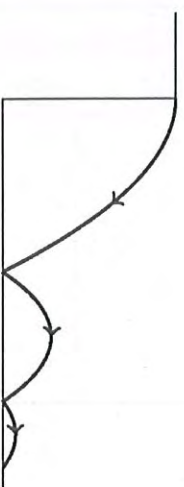
- A. increases
- B. decreases
- C. remains the same
- D. cannot be determined from the data

PHYSICS

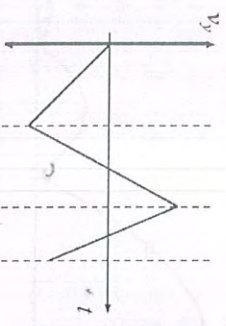
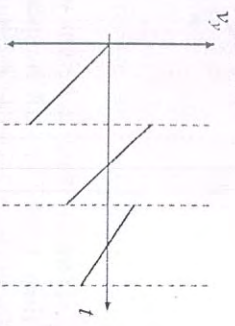
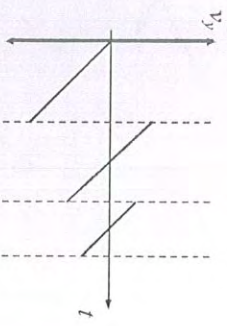
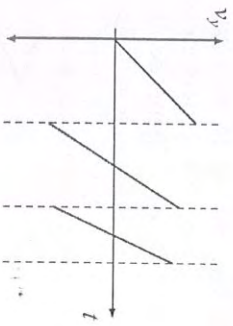
16 A man inside a freely falling box throws a heavy ball towards a side wall. The ball keeps on bouncing between the opposite walls of the box. We neglect air resistance and friction. Which of the following figures depicts the motion of the centre of mass of the entire system (man, the ball and the box)?



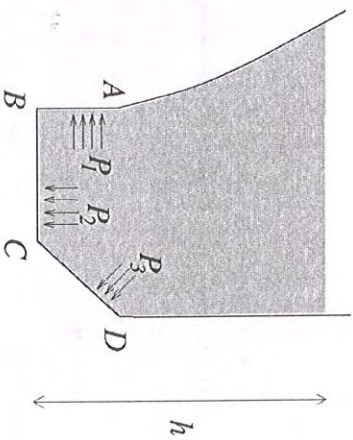
17 A ball is thrown horizontally from a height with a certain initial velocity at time $t=0$. The ball bounces repeatedly from the ground with the coefficient of restitution less than 1 as shown.



Neglecting air resistance and taking the upward direction as positive, which figure qualitatively depicts the vertical component of the ball's velocity (v_y) as a function of time (t)?



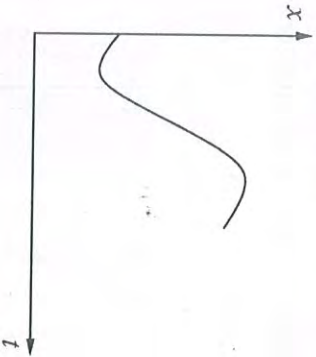
18 A tall tank filled with water has an irregular shape as shown. The wall CD makes an angle of 45° with the horizontal; the wall AB is normal to the base BC . The lengths AB and CD are much smaller than the height h of water (figure not to scale).



Let P_1 , P_2 and P_3 be the pressures exerted by the water on the wall AB , base BC and the wall CD respectively. Density of water is ρ and g is acceleration due to gravity. Then, approximately

- A. $P_1 = P_2 = P_3$
- B. $P_1 = 0, P_3 = \frac{1}{\sqrt{2}} P_2$
- C. $P_1 = P_3 = \frac{1}{\sqrt{2}} P_2$
- D. $P_1 = P_3 = 0, P_2 = h\rho g$

- 19 The accompanying graph of position x versus time t represents the motion of a particle. If p and q are both positive constants, the expression that best describes the acceleration a of the particle is



- A. $a = -p - qt$ B. $a = -p + qt$
 C. $a = p + qt$ D. $a = p - qt$

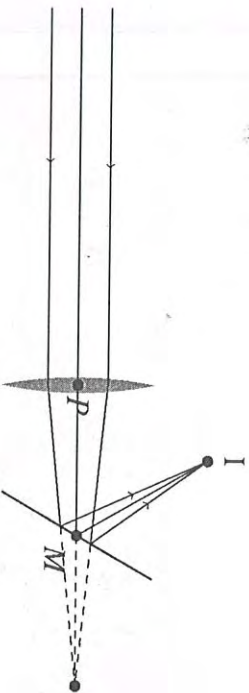
- 20 Two stones of mass m_1 and m_2 (such that $m_1 > m_2$) are dropped Δt time apart from the same height towards the ground. At a later time t the difference in their speed is ΔV and their mutual separation is ΔS . While both stones are in flight
- A. ΔV decreases with time and ΔS increases with time
 B. Both ΔV and ΔS increase with time
 C. ΔV remains constant with time and ΔS decreases with time
 D. ΔV remains constant with time and ΔS increases with time

10

- 21 The refractive index of a prism is measured using three lines of a mercury vapour lamp. If μ_1 , μ_2 and μ_3 are the measured refractive indices for these green, blue and yellow lines respectively, then

- A. $\mu_2 > \mu_3 > \mu_1$ B. $\mu_2 > \mu_1 > \mu_3$
 C. $\mu_3 > \mu_2 > \mu_1$ D. $\mu_1 > \mu_2 > \mu_3$

- 22 A horizontal parallel beam of light passes through a vertical convex lens of focal length 20 cm and is then reflected by a tilted plane mirror so that it converges to a point I. The distance PI is 10 cm.



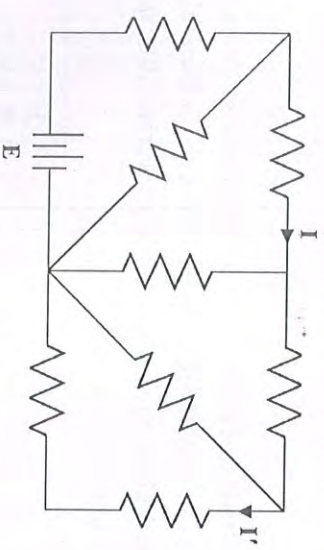
- M is a point at which the axis of the lens intersects the mirror. The distance PM is 10 cm. The angle which the mirror makes with the horizontal is
- A. 15° B. 30°
 C. 45° D. 60°

11

- 23 In a car a rear view mirror having a radius of curvature 1.50 m forms a virtual image of a bus located 10.0 m from the mirror. The factor by which the mirror magnifies the size of the bus is close to

- A. 0.06
B. 0.07
C. 0.08
D. 0.09

- 24 Consider the circuit shown in the figure below:



All the resistors are identical. The ratio I/I' is

- A. 8
B. 6
C. 5
D. 4

- 25 The figure shows a bar magnet and a metallic coil. Consider four situations.

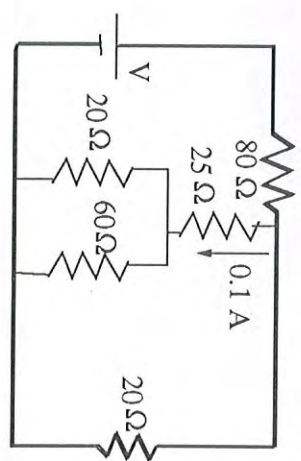
- (I) Moving the magnet away from the coil.
(II) Moving the coil towards the magnet.
(III) Rotating the coil about the vertical diameter.
(IV) Rotating the coil about its axis.



An emf in the coil will be generated for the following situations.

- A. (I) and (II) only
B. (I), (II) and (IV) only
C. (I), (II), and (III) only
D. (I), (II), (III), and (IV)

- 26 A current of 0.1 A flows through a 25 Ω resistor represented by the circuit diagram. The current in the 80 Ω resistor is



- A. 0.1 A
B. 0.2 A
C. 0.3 A
D. 0.4 A

- 27 Solar energy is incident normally on the earth's surface at the rate of about 1.4 kW m^{-2} . The distance between the earth and the sun is $1.5 \times 10^{11} \text{ m}$. Energy (E) and mass (m) are related by Einstein equation $E=mc^2$ where c ($3 \times 10^8 \text{ ms}^{-1}$) is the speed of light in free space. The decrease in the mass of the sun is

- A. 10^9 kg s^{-1}
B. $10^{30} \text{ kg s}^{-1}$
C. $10^{26} \text{ kg s}^{-1}$
D. $10^{11} \text{ kg s}^{-1}$

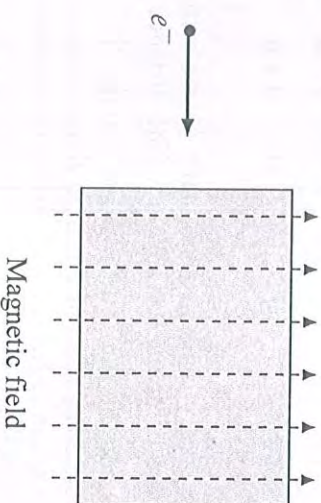
- 28 If the current through a resistor in a circuit increases by 3%, the power dissipated by the resistor

- A. increases approximately by 3%
B. increases approximately by 6%
C. increases approximately by 9%
D. decreases approximately by 3%

29 An ideal gas filled in a cylinder occupies volume V . The gas is compressed isothermally to the volume $V/3$. Now the cylinder valve is opened and the gas is allowed to leak keeping temperature same. What percentage of the number of molecules should escape to bring the pressure in the cylinder back to its original value?

- A. 66% B. 33%
 C. 0.33% D. 0.66%

30 An electron enters a chamber in which a uniform magnetic field is present as shown.



- An electric field of appropriate magnitude is also applied so that the electron travels undeviated without any change in its speed through the chamber. We are ignoring gravity. Then, the direction of the electric field is
- A. opposite to the direction of the magnetic field
 B. opposite to the direction of the electron's motion
 C. normal to the plane of the paper and coming out of the plane of the paper
 D. normal to the plane of the paper and into the plane of the paper

CHEMISTRY

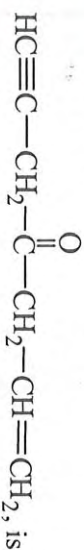
31 The molecule having a formyl group is

- A. acetone B. acetaldehyde
 C. acetic acid D. acetic anhydride

32 The structure of *cis*-3-hexene is

- A.  B. 
 C.  D. 

33 The number of sp^2 hybridized carbon atoms in



- A. 3 B. 5
 C. 4 D. 6

34 The number of valence electrons in an atom with electronic configuration $1s^2 2s^2 2p^6 3s^2 3p^3$ is

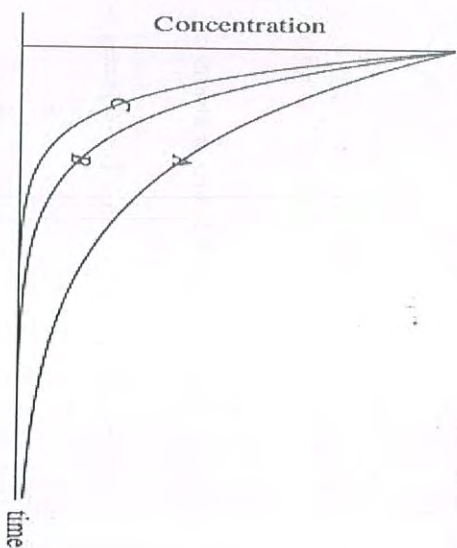
- A. 2 B. 3
 C. 5 D. 11

35 The pair of atoms having the same number of neutrons is

- A. $^{12}_6\text{C}, ^{24}_{12}\text{Mg}$ B. $^{23}_{11}\text{Na}, ^{19}_9\text{F}$
 C. $^{23}_{11}\text{Na}, ^{24}_{12}\text{Mg}$ D. $^{23}_{11}\text{Na}, ^{39}_{19}\text{K}$

- 36 Which of the following molecules has no dipole moment?
- A. CH_3Cl B. CHCl_3
 C. CH_2Cl_2 D. CCl_4

- 37 The decay profiles of three radioactive species A, B and C are given below :



These profiles imply that the decay constants k_A , k_B and k_C follow the order

- A. $k_A > k_B > k_C$ B. $k_A > k_C > k_B$
 C. $k_B > k_A > k_C$ D. $k_C > k_B > k_A$

- 38 A specific volume of H_2 requires 24 s to diffuse out of a container. The time required by an equal volume of O_2 to diffuse out under identical conditions, is
- A. 24 s B. 96 s
 C. 384 s D. 192 s

16

- 39 Acetic acid reacts with sodium metal at room temperature to produce

- A. CO_2 B. H_2
 C. H_2O D. CO

- 40 The equilibrium constant, K_c for



is $4 \text{ L}^2 \text{ mol}^{-2}$. If the equilibrium concentration of benzene is 0.5 mol L^{-1} , that of acetylene in mol L^{-1} must be

- A. 0.025 B. 0.25
 C. 0.05 D. 0.5

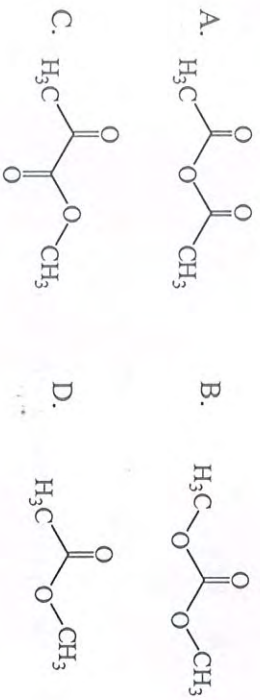
- 41 The weight percent of sucrose (Formula weight = 342 g mol^{-1}) in an aqueous solution is 3.42. The density of the solution is 1 g mL^{-1} , the concentration of sucrose in the solution in mol L^{-1} is

- A. 0.01 B. 0.1
 C. 1.0 D. 10

- 42 The order of reactivity of K, Mg, Au and Zn with water is
- A. $\text{K} > \text{Zn} > \text{Mg} > \text{Au}$ B. $\text{K} > \text{Mg} > \text{Zn} > \text{Au}$
 C. $\text{K} > \text{Au} > \text{Mg} > \text{Zn}$ D. $\text{Au} > \text{Zn} > \text{K} > \text{Mg}$

17

43 Which of the following is an anhydride?



44 Which of the following metals will precipitate copper from copper sulfate solution?

- A. Hg
B. Sn
C. Au
D. Pt

45 The radii of the first Bohr orbit of H (r_H), He^+ (r_{He^+}) and Li^{2+} ($r_{\text{Li}^{2+}}$) are in the order

- A. $r_{\text{He}^+} > r_H > r_{\text{Li}^{2+}}$
B. $r_H < r_{\text{He}^+} < r_{\text{Li}^{2+}}$
C. $r_H > r_{\text{He}^+} > r_{\text{Li}^{2+}}$
D. $r_{\text{He}^+} < r_H < r_{\text{Li}^{2+}}$

BIOLOGY

46 The Bowman's capsule, a part of the kidney is the site of,

- A. filtration of blood constituents
B. re-absorption of water and glucose
C. formation of ammonia
D. formation of urea

47 In human brain the sensation of touch, pain and temperature is controlled by the

- A. parietal lobe of cerebrum
B. limbic lobe of cerebrum
C. temporal lobe of cerebrum
D. frontal lobe of cerebrum

48 A pathogen which can not be cultured in an artificial medium is,

- A. protozoan
B. virus
C. bacterium
D. fungus

49 Meiosis I and Meiosis II are characterised by the separation of,

- A. homologous chromosomes; sister chromatids
B. sister chromatids; homologous chromosomes
C. centromere; telomere
D. telomere; centromere

- 50 People suffering from albinism cannot synthesize
- A. suberin
 - B. melanin
 - C. keratin
 - D. collagen
- 51 Short sightedness in humans can be corrected by using
- A. concave lens
 - B. convex lens
 - C. cylindrical lens
 - D. plain glass
- 52 A person with blood group "A" can (a) donate blood to, and (b) receive blood from,
- A. (a) persons with blood group "AB", and (b) persons with any blood group
 - B. (a) person with blood group "A" or "AB", and (b) "A" or "O" blood groups
 - C. (a) person with blood group "B" or "AB", and (b) "B" or "O" blood groups
 - D. (a) person with any blood group, and (b) "O" blood group only
- 53 Animal cells after removal of nuclei still contained DNA. The source of this DNA is
- A. nucleosomes
 - B. mitochondria
 - C. peroxisomes
 - D. lysosome

- 54 Which one of the following combinations is found in DNA?
- A. Guanine and guanidine
 - B. Guanidine and cytosine
 - C. Guanine and cytosine
 - D. Adenine and guanidine
- 55 Which one of the following is NOT a mode of asexual reproduction?
- A. Binary fission
 - B. Multiple fission
 - C. Budding
 - D. Conjugation

- 56 Which one of the following class of animals constitutes the largest biomass on earth?
- A. Insects
 - B. Fish
 - C. Mammals
 - D. Reptilians
- 57 In the digestive system, the pH of the stomach and the intestine, respectively are,
- A. alkaline; acidic
 - B. acidic; alkaline
 - C. acidic; neutral
 - D. acidic; acidic

58 The major nitrogenous excretory product in mammals is,

- A. amino acids
- B. ammonia
- C. urea
- D. uric acid

59 Which of the following plant traits (characters) is NOT an adaptation to dry (xeric) habitats?

- A. Sunken stomata on leaves
- B. Highly developed root system
- C. Thin epidermis without a cuticle on stem and leaves
- D. Small leaves and photosynthetic stem

60 Biological diversity increases with the productivity of an ecosystem. In which of the following habitats do we see the greatest diversity of species?

- A. Tropical dry grasslands
- B. Temperate deciduous forests
- C. Alpine grasslands
- D. Tropical evergreen forests

PART II
Two-Mark Questions

MATHEMATICS

61 Let a, b, c, d, e be natural numbers in an arithmetic progression such that $a + b + c + d + e$ is the cube of an integer and $b + c + d$ is square of an integer. The least possible value of the number of digits of c is

- A. 2
- B. 3
- C. 4
- D. 5

62 On each face of a cuboid, the sum of its perimeter and its area is written. Among the six numbers so written, there are three distinct numbers and they are 16, 24 and 31. The volume of the cuboid lies between

- A. 7 and 14
- B. 14 and 21
- C. 21 and 28
- D. 28 and 35

63 Let $ABCD$ be a square and let P be a point on segment CD such that $DP:PC=1:2$. Let Q be a point on segment AP such that $\angle BQP = 90^\circ$. Then the ratio of the area of quadrilateral $PQBC$ to the area of the square $ABCD$ is

- A. $\frac{31}{60}$
- B. $\frac{37}{60}$
- C. $\frac{39}{60}$
- D. $\frac{41}{60}$

64 Suppose the height of a pyramid with a square base is

decreased by $p\%$ and the lengths of the sides of its square base are increased by $p\%$ (where $p > 0$). If the volume remains the same, then

- A. $50 < p < 55$
- B. $55 < p < 60$
- C. $60 < p < 65$
- D. $65 < p < 70$

65 There are three kinds of liquids X , Y , Z . Three jars J_1, J_2, J_3 contain 100 ml of liquids X , Y , Z , respectively.

By an operation we mean three steps in the following order:

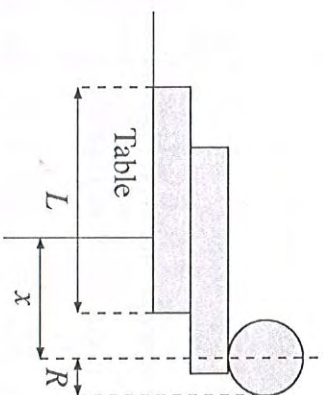
- stir the liquid in J_1 and transfer 10 ml from J_1 into J_2 ;
- stir the liquid in J_2 and transfer 10 ml from J_2 into J_3 ;
- stir the liquid in J_3 and transfer 10 ml from J_3 into J_1 .

After performing the operation four times, let x, y, z be the amounts of X, Y, Z , respectively, in J_1 . Then

- A. $x > y > z$
- B. $x > z > y$
- C. $y > x > z$
- D. $z > x > y$

PHYSICS

66 Two identical uniform rectangular blocks (with longest side L) and a solid sphere of radius R are to be balanced at the edge of a heavy table such that the centre of the sphere remains at the maximum possible horizontal distance from the vertical edge of the table without toppling as indicated in the figure.



If the mass of each block is M and of the sphere is $M/2$, then the maximum distance x that can be achieved is

- A. $8L/15$
- B. $5L/6$
- C. $(3L/4 + R)$
- D. $(7L/15 + R)$

67 Two skaters P and Q are skating towards each other. Skater P throws a ball towards Q every 5 s such that it always leaves her hand with speed 2 ms^{-1} with respect to the ground. Consider two cases:

- (I) P runs with speed 1 ms^{-1} towards Q while Q remains stationary.
 (II) Q runs with speed 1 ms^{-1} towards P while P remains stationary.

Note that irrespective of speed of P , ball always leaves P 's hand with speed 2 ms^{-1} with respect to the ground. Ignore gravity. Balls will be received by Q

- A. one every 2.5 s in case (I) and one every 3.3 s in case (II)
 B. one every 2 s in case (I) and one every 4 s in case (II)
 C. one every 3.3 s in case (I) and one every 2.5 s in case (II)
 D. one every 2.5 s in case (I) and one every 2.5 s in case (II)

68 A 10.0 W electrical heater is used to heat a container filled with 0.5 kg of water. It is found that the temperature of the water and the container rose by 3 K in 15 minutes. The container is then emptied, dried, and filled with 2 kg of an oil. It is now observed that the same heater raises the temperature of the container-oil system by 2 K in 20 minutes. Assuming no other heat losses in any of the processes, the specific heat capacity of the oil is

- A. $2.5 \times 10^3 \text{ JK}^{-1} \text{ kg}^{-1}$
 B. $5.1 \times 10^3 \text{ JK}^{-1} \text{ kg}^{-1}$
 C. $3.0 \times 10^3 \text{ JK}^{-1} \text{ kg}^{-1}$
 D. $1.5 \times 10^3 \text{ JK}^{-1} \text{ kg}^{-1}$

69 A ray of light incident on a transparent sphere at an angle $\pi/4$ and refracted at an angle r , emerges from the sphere after suffering one internal reflection. The total angle of deviation of the ray is

- A. $\frac{3\pi}{2} - 4r$
 B. $\frac{\pi}{2} - 4r$
 C. $\frac{\pi}{4} - r$
 D. $\frac{5\pi}{2} - 4r$

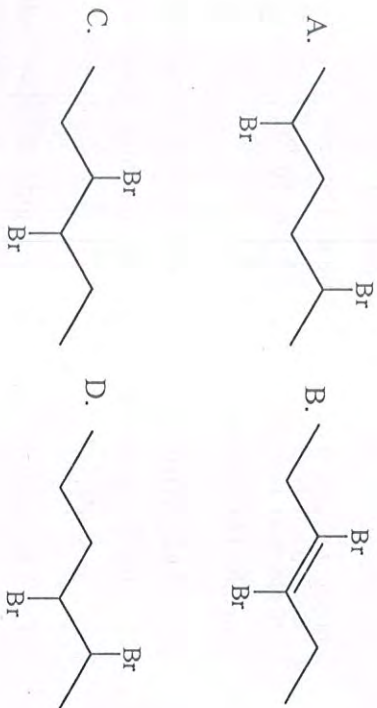
70 An electron with an initial speed of $4.0 \times 10^6 \text{ ms}^{-1}$ is brought to rest by an electric field. The mass and charge of an electron are $9 \times 10^{-31} \text{ kg}$ and $1.6 \times 10^{-19} \text{ C}$, respectively. Identify the correct statement.

- A. The electron moves from a region of lower potential to higher potential through a potential difference of $11.4 \text{ }\mu\text{V}$.
 B. The electron moves from a region of higher potential to lower potential through a potential difference of $11.4 \text{ }\mu\text{V}$.
 C. The electron moves from a region of lower potential to higher potential through a potential difference of 45 V.
 D. The electron moves from a region of higher potential to lower potential through a potential difference of 45 V.

CHEMISTRY

- 71 The degree of dissociation of acetic acid (0.1 mol L^{-1}) in water (K_a of acetic acid is 10^{-5}) is
- A. 0.01 B. 0.5
C. 0.1 D. 1.0

- 72 Compound 'X' on heating with Zn dust gives compound 'Y' which on treatment with O_3 followed by reaction with Zn dust gives propionaldehyde. The structure of 'X' is

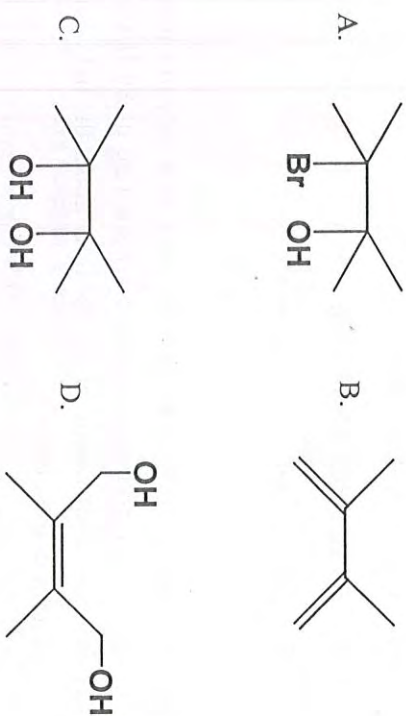


- 73 The amount of metallic Zn (Atomic weight = 65.4) required to react with aqueous sodium hydroxide to produce 1 g of H_2 , is
- A. 32.7 g B. 98.1 g
C. 65.4 g D. 16.3 g

28

- 74 Natural abundances of ^{12}C and ^{13}C isotopes of carbon are 99% and 1%, respectively. Assuming they only contribute to the mol. wt. of C_2F_4 , the percentage of C_2F_4 having a molecular mass of 101 is
- A. 1.98 B. 98
C. 0.198 D. 99

- 75 2,3-Dimethylbut-2-ene when reacted with bromine forms a compound which upon heating with alcoholic KOH produces the following major product



29

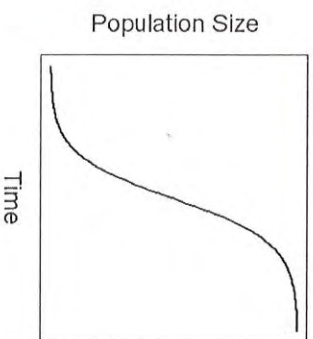
BIOLOGY

- 76 Sister chromatids of a chromosome have
- A. different genes at the same locus
 - B. different alleles of the same gene at the same locus
 - C. same alleles of the same gene at the same locus
 - D. same alleles at different loci
- 77 A diabetic individual becomes unconscious after self-administering insulin. What should be done immediately to revive the individual?
- A. Provide him sugar
 - B. Give him higher dose of insulin
 - C. Provide him salt solution
 - D. Provide him lots of water
- 78 A regular check on the unborn baby of a lady towards the end of her pregnancy showed a heart rate of 80 beats per minute. What would the doctor infer about the baby's heart condition from this?
- A. Normal heart rate
 - B. Faster heart rate
 - C. Slower heart rate
 - D. Defective brain function

- 79 Three uniformly watered plants i, ii and iii were kept in 45% relative humidity, 45% relative humidity with blowing wind and 95% relative humidity, respectively. Arrange these plants in the order (fastest to slowest) in which they will dry up.

- A. i = ii, iii
- B. ii, i, iii
- C. iii, ii, i
- D. iii, i = ii

- 80 Many populations colonising a new habitat show a logistic population growth pattern over time, as shown in the figure below.



- In such a population, the POPULATION growth rate
- A. stays constant over time
 - B. increases and then reaches an asymptote
 - C. decreases over time
 - D. increases to a maximum and then decreases

ROUGH WORK